Evaluating Corporate Bonds and Analyzing Claim Holders' Decisions with Complex Debt Structure*

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Abstract

Though many studies elucidate how an issuing firm's investment and financing policies influence its claim holders' values via structural credit models, most of such models still fail to capture phenomena found in financial markets due to oversimplification of the firm's debt structure. This paper proposes a quantitative framework to model a typical complex debt structure containing multiple bonds with various covenants. Rather than relying on naive settings on default triggers and future financing policies, our framework models a firm's insolvency risk through the default trigger shaped according to the characteristics of its debt structure, like the amount and the schedule of bond repayments. Thus it can provide theoretical insights and concrete quantitative measurements consistent with extant empirical researches, like the shapes of yield spread curves under different issuer's financial statuses, and the impacts of including payment blockage covenants on newly issued and other outstanding bonds. We also develop a novel quantitative method, the forest, to handle the contingent changes of the debt structure due to premature bond redemptions. A forest consists of several trees that captures different debt structures, says before or after a bond redemption. This method can analyze how the poison put covenants in the target firm's bonds influence the bidder's costs of debt financing for a leveraged buyout, investigate how the presence of wealth transfer among the remaining claim holders due to a bond redemption influences the firm's call policy, and further reconcile the conflicts among previous empirical studies on call delay phenomena.

Keywords: debt-structure-dependent default trigger, forest, payment blockage covenants, poison put covenant, call policy, wealth transfer effect

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1 Introduction

Corporate bonds are fundamental financing instruments that are widely held by institutional investors or fund managers. According to the reports of Securities Industry and Financial Markets Association (SIFMA), the amount of issuances (outstandings) in the US market grows from 343.7 billion (2247.9 billion) in 1996 to 1434.8 billion (7846.2 billion) in 2014.[1] This entails that a corporate bond is playing an important role in capital markets, and its prevalence further makes academics and practitioner communities pay more attention to analyzing bond evaluations and relevant claim holders' decisions (e.g., early redemptions of bonds).

While a *default-free* bond (e.g. a Treasury bond) can be *separately* evaluated without considering the presence of other simultaneously outstanding default-free bonds, the value of a corporate bond may be greatly influenced by the existence of other outstanding bonds of the same issuer due to the claim dilution effect. For example, Fama and Miller (1972) indicate that the new bond issuances may dilute the values of other previously issued bonds. Ingersoll (1987) further points out that the issuances of short-term junior bonds may deteriorate the credit quality of the previously issued long-term senior bonds. Indeed, the empirical investigations in Rauh and Sufi (2010) and Colla et al. (2013) identify that most corporate bond issuers have very complicated debt structures, like multiple outstanding bonds with different maturities, priorities and embedded covenants. To analyze the relationships among an issuer's debt structure, the prices of its outstanding bonds/equities, and relevant claim holders' decisions, we construct a quantitative framework that endogenously associates the issuer's insolvency risk with its prevailing debt structure by taking advantage of the structural model pioneered by Merton (1974). This framework provides theoretical insights and concrete quantitative measurements for the empirical literature on debt structure.

To reduce mathematical or computational difficulty for modeling complex features of an issuer's debt structure, many structural models oversimplify the debt structure and perform poorly for evaluating corporate securities as examined in Jones et al., 1984, Eom et al. (2004) and Huang and Huang (2012). For example, some models use a "representative bond" to stand for the overall complex debt structure (e.g. Merton (1974), Kim et al. (1993) and Leland (1994)) and this simplification prevent us from analyzing the impacts of coexisting bonds with different covenants on the values of the issuer's securities. Another popular approach, the "portfolio of zeroes approach", decomposes all outstanding bonds of the same issuer into a portfolio of equal-priority zero-coupon bonds and evaluates them separately (e.g. Longstaff and Schwartz (1995) and Collin-Dufresne and Goldstein (2001a)). Eom et al. (2004) indicate that this approach would inaccurately estimate the default probability of each zero-coupon bond since each bond is evaluated without considering whether all previously matured bonds are honored or not. In addition, many models preserve mathematical tractability by putting naive settings on default triggers. For example, Black and Cox (1976) and Zhou (2001) assume that the issuing firm defaults when its asset value falls below an unified default boundary without considering the debt repayment schedule defined in the firm's debt structure. Other works consider repayment schedules with naive financing settings. For example, Geske (1977) assumes that all debt repayments are financed by issuing new equities. Leland and Toft (1996) assume that the firm should keep the amounts of outstanding bonds unchanged regardless of its financial status on repayment dates. However, Davydenko (2012) empirically shows that it is hard to identify a unified boundary level to exactly separate insolvent issuers from solvent ones, and many empirical evidences confirm that an issuer's refinancing policy may depend on its current financial status, its investment opportunities, or the

[1]See http://www.sifma.org/research/statistics.aspx

macroeconomic condition.[2]

Much effort has been devoted to enhancing empirical validity of structural models. For examples, Eom et al. (2004) empirically show that the "compound option approach" is much better than the aforementioned portfolio of zeros approach. Specifically, the former approach views whether a due bond principal or coupon repayment is honored or not as an option on other options — whether previously matured repayments are fulfilled or not. Thus the default probability for each repayment is evaluated conditionally on the default probabilities for previous repayments in the former approach, and this is more reasonable than modeling default events independently in the latter approach. Besides, some papers elaborate structural models by considering the interdependence of an issuer's investment policies and different facets of its debt structure, like bond maturities (e.g., Barclay et al. (2003)), priorities (e.g., Hackbarth and Mauer (2012)), and leverage ratios (e.g., Kuehn and Schmid (2014)).

To appropriately associate an issuer's insolvency risk with different observable facets of its debt structure based on the compound option approach, we develop a novel evaluation framework by taking the advantage of a popular numerical technique, the tree method, proposed by Cox et al. (1979). Through the flexibility of the tree method, our framework can easily model the debt-structure-dependent default trigger shaped according to the payment schedule and covenants embedded in the issuer's outstanding bonds. and this provides a theoretical insight into Davydenko (2012)'s observation that default triggers are widely dispersed among firms. Specifically, to measure an issuer's ability to repay a certain obligation with its internal or external funds under the burdens of previously matured payments, we introduce a novel proxy, "remaining asset", which is defined as the remaining of the firm asset value after repaying all required bonds matured prior to that obligation. This proxy allows our framework to implicitly incorporate the compound option approach's spirit without adopting naive financing settings,[3] says, financing all repayments by raising new equities as in Geske (1977) or keeping the amounts of outstanding bonds stationary as in Leland and Toft (1996). To model the influence of payment schedules and covenants, a default event is triggered once the issuer's remaining asset value minus the values of the assets pledged for other outstanding bonds is less than its matured payment defined in its debt structure. Though introducing the concepts of remaining assets and the "debt-structure-dependent" default trigger makes the resulting mathematical model complicated as mentioned in Section 2, the flexibility of our framework can overcome these difficulties to provide reliable evaluations and theoretical insights into many empirical studies.

To demonstrate how simplifying debt structures and adopting naive financing settings would generate inaccurate bond evaluation results, **Fig. 1** illustrates the yield spread curves for simultaneously issued bonds (i.e., serial bonds) of the same issuer extracted from empirical data and those generated from different structural models. The empirical studies in Helwege and Turner (1999) and Huang and Zhang (2008) suggest that most yield spread curves implied by serial bonds are upward-sloping despite of the financial status of the issuing firm.[4] Two typical examples of the yield spread curves

[2]For example, Barclay et al. (2003) indicate that an issuing firm's choices of bond maturity are closely related to its investment opportunity. Besides, Rauh and Sufi (2010) and Hackbarth and Mauer (2012) show that unhealthy issuing firms may spread priority across debt classes. In addition, Chen et al. (2013) show that firms with high systematic risk favor long-term bond issuances and will have more stable debt maturity structure over the business cycle. Xu (2014) shows that speculative-grade firms are actively extending their debt maturity structure in good times. Similarly, Kahl et al. (2015) shows that, instead of keeping using short-term bonds like commercial papers, firms with high rollover risk often issue long-term bonds to replace the maturing short-term bonds.

[3]Gopalan et al. (2014) proposes that the refinancing policy for the bonds that are about to mature (i.e., bonds maturing within 1 year) are likely to be certain. Our evaluation framework can also deal with this scenario as discussed in Section 4.1.2.

[4]The empirical results in Helwege and Turner (1999) show that, in primary market, over 80% of the yield spread curves implied by those equal-priority bonds issued on the same day by the same speculative-grade issuer are upward-

for an investment-grade issuer DIRECTV Holdings (in the black curve) and a speculative-grade issuer Rockies Express Pipeline (in the gray curve) are illustrated in **Fig. 1 (a)** to demonstrate this upward-sloping nature. Then we examine the reliability of different structural models by evaluating the issuing prices of three otherwise identical serial bonds of a hypothetical issuer with maturities 5, 10, and 20 years. In **Fig. 1 (b)**, these three serial bonds are either evaluated separately without considering the presence of other bonds (denoted by dashed curves) or by our quantitative framework (denoted by solid curves). As noted by Jones et al. (1984) and Kim et al. (1993), the former setting ignores the impacts of coexisting bonds and overly underestimates the bond yield spreads when the issuing firm is healthy as plotted in the black dashed curve. Furthermore, when the firm's creditworthiness deteriorates, the former setting generates a hump-shaped (plotted in light gray dashed) yield spread curve or a downward-sloping (plotted in dark gray dashed) one; these shapes are inconsistent with the upward-sloping nature found in empirical results. In contrast, our framework can analyze how the repayments of short-term bonds deteriorate the firm's solvency to further jeopardize the credit quality of the long-term bonds. Thus, it would not significantly underestimate the bond yield spreads for a healthy issuer (plotted in the black solid curve) and generate reasonable upward-sloping yield spread curves regardless the issuing firm's financial status.

To alleviate mathematical or computational difficulty for modeling complex debt structures, much literature adopts naive settings on default triggers and financing strategies. Three typical simplified settings adopted widely in extant literature are examined under the aforementioned hypothetical scenario as illustrated in **Fig. 1 (c)**, **(d)** and **(e)**. The **(S.1)** setting adopts an unified default boundary without considering repayment schedule implied by the issuer's debt structure (e.g. Black and Cox (1976)). The **(S.2)** setting assumes that all future loan repayments are only financed by raising new equities (e.g. Geske (1977)). The **(S.3)** setting assumes that the issuing firm keeps its debt structure unchanged without considering its prevailing financial status (e.g. Leland and Toft (1996)).[5] Unlike the upward-sloping solid yield spread curves generated by our framework, adopting **(S.1)**, **(S.2)** and **(S.3)** could lead to infeasible hump-shaped or downward-sloping curves as plotted by dash curves denoted in **Fig. 1 (c)**, **(d)** and **(e)**, respectively.

To overcome mathematical intractability for modeling complex debt structures, Broadie and Kaya (2007) and Wang et al. (2014) take advantage of the flexibility of the tree method. The former work considers the impacts of reorganization processes under Chapter 11 of US bankruptcy code and the complex tree implementation would lead to unstable pricing results due to nonlinearity errors (see Figlewski and Gao, 1999).[6] The latter work studies how to adjust the tree structure to stably evaluate multiple outstanding bonds of the same issuing firm. By incorporating the aforementioned consideration of the repayment schedule and the default trigger implied by the issuer's debt structure, our framework can not only provide concrete quantitative measurements for the phenomena identified in extant empirical literature but give theoretical insights into these observations, like reliable shapes of yield spread curves in **Fig. 1**. In addition, the robustness of our framework are examined by comparing our quantitative results with relevant empirical studies, like the magnitude of the yield spread influenced by the level of the interest rate (see Duffee (1998)), the firm value volatility Avramov

sloping; over 60% of the yield spread curves for the cases in secondary market are also upward-sloping. The empirical investigation into the broader sample sets by Huang and Zhang (2008) displays that more than 80% of the yield spread curves for the cases of investment- and speculative-grade issuers are upward-sloping.

[5]The details of these three settings will be discussed in Section 2.2.2.

[6]Broadie and Kaya (2007) demonstrate their oscillating numerical results in Fig. 5, 6, and 8. Indeed, analyzing the impacts of various reorganization procedures on the benefits and decisions of different claim holders without interfere from numerical errors can be an important topic for future studies.

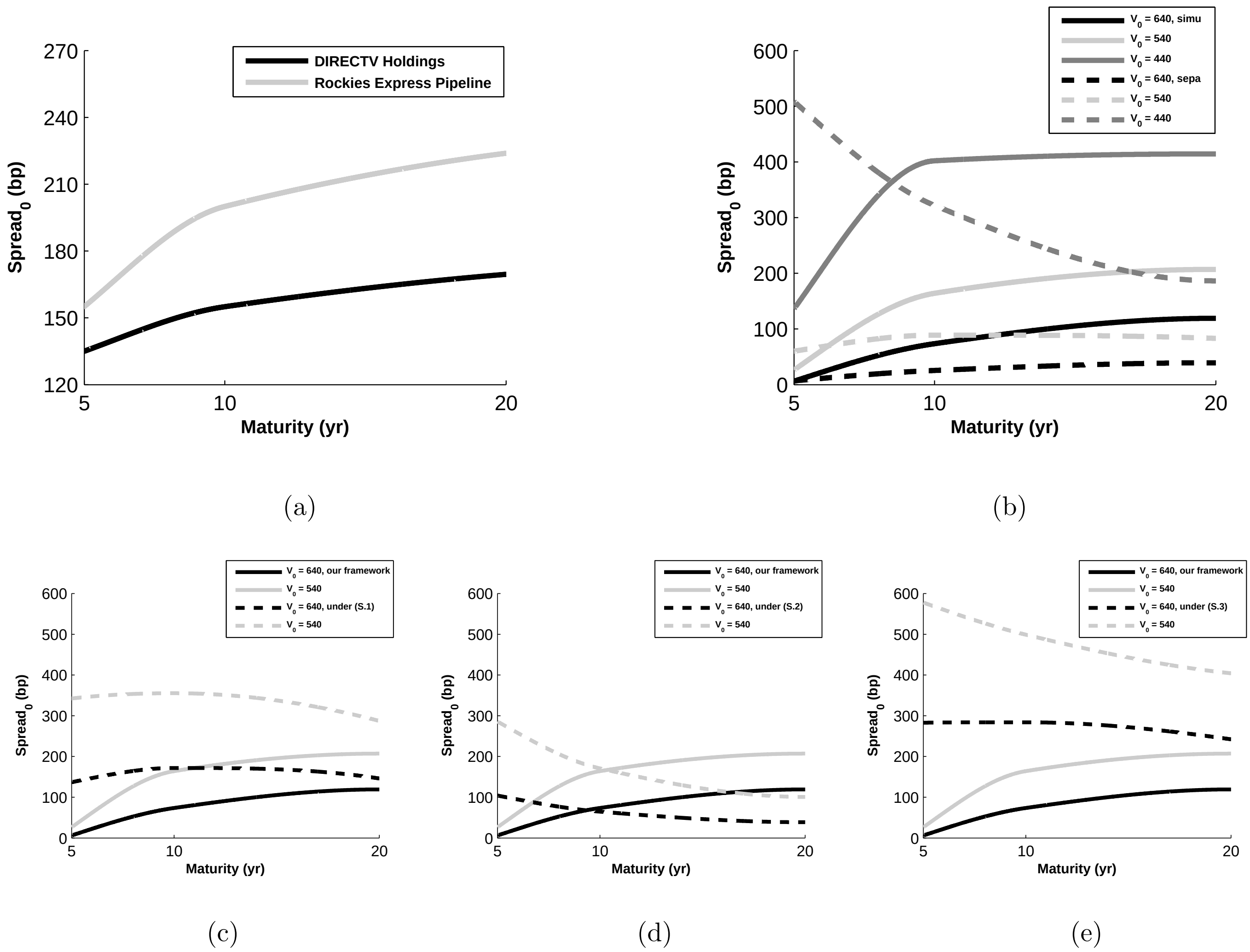


Figure 1: Shapes of yield spread curves. Panel (a) illustrates the yield spread curves implied by equal-priority bonds issued on the same day by an investment-grade issuer, DIRECTV Holdings (TRACE CUSIP: 25459HAY1, 25459HBA2, 25459HAZ8) and a speculative-grade issuer, Rockies Express Pipeline (TRACE CUSIP: U75111AE1, U75111AF8, U75111AG6). The data is from Mergent Fixed Income Securities Database. The following panels illustrates how the structural models with different settings can generate varied yield spread curves of three otherwise identical par bonds issued by a hypothetical firm with maturities 5, 10 and 20 years. The risk-free interest rate is 2%, the asset volatility of the firm is 20%, the lump sum of the face values of these three bonds is 300, the tax rate is 35%, and the loss ratio of the firm asset value due to liquidation is 50%. The level of the firm asset value at its issuance date V_0 is used as a proxy for the firm's prevailing financial status. The black and gray colors denote that the prevailing firm's financial statuses are good and poor, respectively. The solid curves denote the yield spread curves generated by our framework. The dash curves in panel (b) are generated by evaluating these three bonds separately. The dash curves in (c), (d), (e) are generated by extant structural models with settings **(S.1)**, **(S.2)**, **and (S.3)**.

et al. (2007)), and the leverage ratio (see Collin-Dufresne et al., (2001b) and Flannery et al. (2012)).[7] Our framework can also model the effects for rolling over matured bonds or callable bonds as in Section 4.1.2 and Appendix A.1 to compare with relevant studies like Gopalan et al. (2014) and Nagler (2014). Note that some covenants, like payment blockage covenants and call provisions, may change the order

[7]See Appendix B and C for details.

for repaying outstanding bonds and hence their credit qualities. The following discussions show how our framework quantitatively models these covenants to provide theoretical insights and to explain the conflicts in past empirical studies.

Linn and Stock (2005) empirically study the impact of a junior bond issuance on the previously issued senior bond. They find that whether the junior bond is matured prior to the senior bond's maturity or not does not have salient difference on the claim dilution effect as predicted in Ingersoll (1987), who argues the payment schedule also implicity influences the seniority of bonds. The reason might be that the holders of a latter-matured senior bond are usually granted limited rights to block certain payments to former-matured junior bonds to ensure their payments are fully repaid. Incorporating payment blockage covenants into our framework allow us to duplicate and to explain the phenomena observed in Linn and Stock (2005).

To capture contingent changes of a debt structure due to early redemptions of callable or putable bonds, a novel quantitative method named the "forest" is developed by systematically combining several trees arranged in layers; each tree captures one possible scenario of the debt structure, says before or after a bond early redemption. This design is because each early redemption can change the order of bond repayments and accordingly the default trigger, thus leading to wealth transfer among the firm's remaining claim holders. The forest method is, to our knowledge, the first quantitative method that can analyze complicated call policies of multiple outstanding callable bonds issued by the same firm, which is an open problem addressed in Jones et al. (1983).[8] The ability to evaluate the wealth transfer effect can be used to analyze call delay phenomena, the phenomena that a callable bond will not be redeemed until its price far exceeds the call price. Our framework shows that the significance of the wealth transfer effect is influenced by the levels of interest rates, the magnitudes of call prices, the length of call protection periods, and the remaining time to maturities of callable bonds. These studies help us to reconcile the conflict between Longstaff and Tuckman (1994) and King and Mauer (2000) on the relationship between wealth transfer effects and call delay phenomena. Besides, our framework can also analyze how the wealth transfer effect due to exercising poison put covenants embedded in target firm's bonds decreases yield spreads of these bonds at the expense of bidder's costs of debt financing for a leveraged buyout as studied in Cremers et al. (2007).

The rest of the paper is organized as follows. In Section 2, we describe our valuation framework, including the concepts of the remaining asset and the debt-structure-dependent default trigger. In Section 3, we first elaborate the method in Wang et al. (2014) to implement the payment blockage covenant. Then we describe how a forest is developed to capture contingent changes of an issuer's debt structure due to premature redemptions. Section 4 illustrates how our framework provides concrete quantitative measures as well as theoretical insights to aforementioned empirical studies and further reconcile the conflicts in these studies. Section 5 concludes this paper.

2 Model Settings

2.1 Fundamentals Settings of Structural Models

A structural model specifies the evolution of the market value of the issuing firm's asset and the conditions leading to default. We follow Merton (1974) by assuming that the asset value at an arbitrary time t, V_t, obeys the following lognormal diffusion process under the risk-neutral probability

[8] Jones et al. (1983) present a system of partial differential equations with complicated boundary conditions without solving the problem.

measure:

$$dV_t = rV_t dt - P_t + \sigma V_t dz. \tag{1}$$

Here we follow Attaoui and Poncet (2013) by setting r as the long-term average interest rate since the impact of stochastic interest rate can be negligible as suggested in Ju and Ou-Yang (2006). P_t denotes the payout for dividend or debt repayments at time t. It depends on different settings of future financing strategies adopted in existent structural models and will be discussed in Section 2.2. σ denotes the volatility and can be viewed as a proxy for the firm's business risk. We follow Fan and Sundaresan (2000) by setting σ as a constant since the firm manager cannot alter the business risk arbitrarily due to restrictive covenants embedded in outstanding bonds of the firm. dz denotes a standard Brownian motion.

To meet investment and finance requirements at different periods of time, a firm would issue multiple bonds with different maturities and covenants at different time points. Thus we can model a firm's debt structure to be comprised of N outstanding bonds: $B_1,\ldots, B_N$. The face value, the annual coupon payment, and the time to maturity for the *i-th* bond B_i are denoted as F_i, C_i, and T_i, respectively. We set $0 < T_1 <\ldots< T_N = T$ for ease of later discussions. In a structural model, all bonds and the leveraged equity of the same issuing firm can be viewed as contingent claims on the firm's asset. For convenience, the values of these contingent claims at time t are denoted as $B_1(t, V_t)$, $\ldots$, $B_N(t, V_t)$ and $E(t, V_t)$. To model the firm's value gained from tax shield benefits by using debt capital, we assume that the firm's coupon payments are tax-deductible at an exogenous tax rate τ, $\tau \in (0, 1)$. On the other hand, raising debt capital incurs a proportional cost of k, $k \in (0, 1)$, which is expressed as a fraction of the market value of the newly issued bond (see Chen, 2010).

Despite tax saving benefits, using debt capital would also incur bankruptcy cost resulting from costs of liquidation due to inability to fulfill debt repayments. The key difference between default triggers in our framework and most past structural models is the incorporation of the payment schedule defined in the issuing firm's debt structure. This schedule is in practice an important factor for analyzing issuers' default risks.[9] Many past structural models adopt unified default boundaries without considering the burdens due to scheduled loan repayments as discussed in setting **(S.1)** in Section 2.2.2. On the other hand, our framework follows Moody's definition of debt default as a firm misses a disbursement of a contractually-obligated interest or principal payment defined in bond indentures (see Ou et al., 2011). Besides, to make our paper concisely analyze the disadvantages of oversimplifications of debt structures and adoptions of naive financing strategies, all structural models in this paper are analyzed based on the Chapter 7 proceedings as in Kuehn and Schmid (2014); that is, an insolvent firm is liquidated immediately after filing for bankruptcy. A constant fraction ω, $\omega \in (0, 1)$, of the firm's asset value is lost as liquidation costs, like the legal fees (see Leland, 1994). Leftover assets are then distributed according to the absolute priority rule as the evidence reported in Bris et al. (2006).[10]

As fund suppliers, both bonds and equities investors want to assure themselves of getting fair returns on their investments. Generally speaking, a firm manager acts for equity holders and tends to maximize their benefits at the expense of bond holders (see Jensen and Meckling, 1976; Myers, 1977). This observation can be used to model an issuer's decisions, says, an optimal strategy for early redemption of callable bonds analyzed in Section 4.3. To alleviate this agency problem (see Smith and

[9] For example, Greece debt crisis is analyzed with its payment schedule in The Economist website http://www.economist.com/blogs/graphicdetail/2015/04/daily-chart-7 and many other related articles.

[10] Note that reorganizations under Chapter 11 of US bankruptcy code are also widely studied (e.g. François and Morellec (2004), Broadie et al. (2007) and Galai et al. (2007)). Implementing the Chapter 11 on the tree method is preliminarily studied in Broadie and Kaya (2007) and incorporating their implementation into our framework can be a good topic for future studies.

Warner, 1979), bond investors will require collateral pledged for their bonds or restrictive covenants to ensure security of their claims. Assets pledged as collateral influence the solvency of the firm and this factor is modeled by the debt-structure-dependent default trigger in our framework. A covenant protects its host bond value either by restricting firm managers's behaviors (e.g. issuing new bonds) or by granting host bond holders the right to change the orders of repayments among different claims. Changing repayment orders causes wealth transfer among holders of all outstanding claims and hence influence the investors' decisions for enforcing covenants. To explain relevant empirical phenomena analyzed in past studies and reconcile the conflicts among these studies, our framework quantitatively analyzes two such covenants, the payment blockage and the poison put, plus the call provision that grants firm managers similar rights. These three covenants are introduced as follows.

A payment blockage covenant grants senior bond holders the right to block scheduled payments to later issued junior bonds in order to ensure that their payments are fully repaid before the payments to the junior bonds (see Linn and Stock, 2005; Davydenko, 2007). This covenant can avoid the presence of payments to short-term junior bond holders from jeopardizing the effective seniority of long-term senior bonds. It can block all the payments to junior bond holders during the blockage period $[t^* - \eta, t^*]$[11] to satisfy payments to senior bond holders if the firm defaults at time t^*. In Section 4.2, we will show that implementing this covenant can explains the insignificant claim dilution effects of existing senior bonds due to issuances of short-term junior bonds found in Linn and Stock (2005)

Poison put covenants grant bond holders the right to sell the bonds back to the firm prematurely at a predetermined put price (abbreviated as `PP`) due to occurrences of predetermined unfavorable events, such as a leveraged buyout (abbreviated as `LBO`).[12] Typically, the `PP` is equal to or above the bond face value. In Section 4.4, we will analyze how wealth transfer effect due to enforcing poison put covenants can protect holders of target firm's bonds at the expense of bidder's costs of debt financing for a `LBO` studied in Cremers et al. (2007).

Call provisions grant the issuer the right to redeem the host bond prematurely at an effective call price (abbreviated as `CP`), which is defined as the call price predetermined in the bond's prospectus plus the accrued interest (see Thatcher, 1985). Callable bonds usually contain the covenant of the call protection period, a period that the bond is protected from being called, to protect interests of bond investors. In Section 3.3, we will develop a novel numerical method, a forest method, that can analyze complicated call policies of multiple outstanding callable bonds. The relationships between wealth transfer effect due to premature redemptions and call delay phenomena that are widely examined by empirical literature will be quantitatively analyzed in Section 4.3.

The bond value for B_i at time 0, denoted as $B_i(0, V_0)$, can be evaluated by different structural models introduced in the next subsection. The yield to maturity for B_i, denoted as $Y_0^{B_i}$, would make the bond value equal the lump sum of discounted future cash flows as follows.

$$B_i(0, V_0) = F_i e^{-T_i Y_0^{B_i}} + \sum_{j=1}^{nT_i} \frac{C_i}{n} e^{-\frac{j}{n} Y_0^{B_i}}, \tag{2}$$

where n denotes the frequency of coupon payments per year. The corresponding yield spread $\text{Spread}_0^{B_i}$

[11]The length of blockage period η usually ranges from 90 days to a year or more. See http://www.uccstuff.com/CLASS NOTES/SubordinatedDebt.shtml.

[12]Other unfavorable events include decapitalizations, recapitalizations, restructurings, mergers, acquisitions, share repurchases, increment of leverage ratio, or credit rating downgrading.

can then be derived by substituting $Y_0^{B_i}$ solved in Eq. (2) into the following equation:

$$\text{Spread}_0^{B_i} = Y_0^{B_i} - r.$$

2.2 Settings Adopted in Structural Models

2.2.1 Our Framework: the Remaining Assets and Debt-Structure-Dependent Default Trigger

The key point that a compound option approach performs better as mentioned in Eom et al. (2004) might be due to the fact that this approach considers the impacts of repayments of coexistent outstanding bonds on other unmatured bonds. Note that bond repayments can be financed by either internal funds or external ones, such as raising new equities or bonds. Thus financing decisions on repayments and hence default triggers are uncertain especially when the repayments are far into the future. Since adopting naive assumptions on financing decisions and default triggers would lead to inaccurate shapes of yield curves as in **Fig. 1**, our framework evaluates an issuing firm's bonds or equities with two novel concepts, remaining assets and debt-structure-dependent default triggers, to avoid adopting any assumptions on future financing decisions.

Remaining assets can be viewed as a proxy for measuring a firm's ability to repay a certain obligation, says bond B_i's principal F_i at time T_i, under the burdens of previously matured payments. It is defined as the *remaining* of the firm asset value after repaying every required payment occurred before time T_i. Specifically, let C_t^O denote the repayment amount occurred at time t defined in the debt structure.[13] Then the process of remaining assets is constructed by defining P_t in Eq. (1) as $C_t^O dS_t$, where S_t is a step function that increases by one at each repayment date. Therefore, the process of remaining assets follows a lognormal diffusion process between two adjacent payment dates and decreases with the size C_t^O at time t; in other words, the remaining asset value at a payment time t, V_t, can be expressed as $V_{t^-} - C_t^O$, where t^- denotes the time immediately before time t. The value decrements reflect the burdens of previous repayments and potentially lower the magnitude of V_{T_i}, which indicates a lower level of internal funds and poorer ability to raise enough external funds (due to the low equity value or the debt overhang problem) to finance the repayment F_i at time T_i.

We follow Moody's definition (see Ou et al., 2011) by defining a default event as issuing firm's inability to fulfill repayments defined in its debt structure. Specifically, default occurs at a payment time t once the level of its remaining asset V_{t^-} minus Λ_t, the frozen assets at time t due to restrictive covenants like collateral of secured bonds, can not meet the repayment C_t^O; in other words, the firm defaults if V_{t^-} is lower than a debt structure-dependent default boundary Θ_t, which is defined as $C_t^O + \Lambda_t$. Note that our framework is analogous to the compound option approach since whether each repayment is serviced or not depends on whether its previous payments and covenants are honored or not.

Besides, the issuance or rollover strategies in the near future are usually planned to be certain and the impacts for these strategies on yield spreads are also widely studied empirically. Our framework can incorporate these impacts by introducing the issuance cost k into the firm value process and analyze empirical phenomena found in relevant literature in Section 4.1. For example, issuing new claims with market value C_t^I at time t for investment purposes would change the leverage ratio and

[13]This paper do not consider dividend payments or other disbursements distributed to equity holders as in Ingersoll (1977a) to make our analyses focus on the relationship between required payments and the firm's solvency. Actually, previous empirical literature (e.g., Collin-Dufresne et al. (2001b); Avramov et al. (2007)) also does not consider dividend payments as explanatory variables of bond yield spreads.

result in the claim dilution effect as in Flannery et al. (2012). Our framework can evaluate all claims (including newly issued claims) as derivatives on V_t, whose value is equal to the pre-issuance firm asset value plus the net-of-cost proceeds from issuance: $V_{t^-} + (1-k)C_t^I$. Gopalan et al. (2014) study the rollover risk for financing C_t^O by issuing new bonds with market value C_t^I and our framework would adjust the after rollover firm value V_t as $V_{t^-} - C_t^O + (1-k)C_t^I$.

Prematurely redeeming a callable bond would change its prevailing debt structure and hence the payment schedule as well as the default trigger. Callable bonds tend to be redeemed at an optimal stopping time to maximize the benefits of equity holders and the redemptions would redistribute wealth among remaining claim holders. To determine whether it is optimal or not to redeem a callable bond, two trees are required to evaluate the equity values before and after the redemption, respectively. The former tree evaluates all claims (including the callable bond) based on the debt structure that contains all unmatured payments of that callable bond. The latter tree evaluates all other claims (excluding the callable bond) based on the debt structure without that callable bond. The callable bond will be redeemed early if the equity value calculated in the latter tree is higher than its corresponding value in the former tree. Note that a early redemption transfers the firm's status from the former tree to the latter one, which can be modeled by a novel numerical method, forest, that combines these two tress as mentioned in Section 3.3.

2.2.2 Comparison with Other Frameworks

Instead of modeling a sophisticating designed default boundary based on the payment schedule and restrictive covenants defined in the firm's debt structure as mentioned above, most exogenous default boundary models[14] exogenously specify "unified" default boundaries (see Davydenko, 2012) to preserve mathematically tractability as follows.

> **(S.1)** The default is triggered once the firm's asset value is lower than an exogenously given unified boundary constructed with simplified assumptions on covenants.

For example, many models adopt safety covenants (e.g., Black and Cox, 1976), maintenance covenants (e.g., Longstaff and Schwartz (1995)), or other similar ones to specify unified default boundaries such as (the discounted value of) exogenously given constants (e.g. Black and Cox (1976), Kim et al. (1993), and Longstaff and Schwartz (1995)), (a fraction of) market values of outstanding bonds (e.g., Briys and De Varenne (1997) and Ju and Ou-Yang (2006)), the face value of the firm's short-term bonds plus half of the face value of the long-term bonds (e.g., Crosbie and Bohn (2002)), 66% of the firm's total bond face value (e.g., Davydenko (2012)), etc. Indeed, Davydenko (2012) notes that this type of covenants is rarely included in bond indentures in practice; therefore, these covenants should be interpreted as simplified proxies for complex restrictive covenants defined in the issuer's debt structure. However, he further addresses that it is difficult to specify an unified boundary level to exactly separate insolvent firms from solvent ones, since the empirically observed boundaries are widely dispersed among firms. Thus, even these structural models can calibrate their unified default boundaries to perform reasonably well on average, relying on unified default boundaries may still contribute to poor cross-sectional prediction accuracy for bond yield spreads (see Eom et al., 2004). For example, by following Davydenko (2012)'s suggestion to set the default boundary as 66% of the lump sum of all outstanding bonds' face values, improper hump-shaped yield spread curves are generated as illustrated by the dashed curves in **Fig. 1 (c)**.

[14] Leland (2004) categorized default boundaries in structural models into two types: exogenous default boundaries and endogenous ones.

Some other structural models determine their default boundaries endogenously to maximize equity values (e.g., Leland (1994)) or to ensure the firm's free cash flow to be able to cover interest expenses (e.g., Attaoui and Poncet (2013)). Financing strategies for obligated loan repayments significantly influence the behaviors and reliabilities of structural models. Two simplified strategies that are widely adopted are listed as follows.

- **(S.2)** Geske (1977) assumes that each bond repayment C_t^O occurred at a payment date t is simultaneously financed by issuing new equities without changing the firm's asset value; that is, $P_t = (C_t^O - C_t^I)dS_t = 0$ in Eq. (1)). The firm files for bankruptcy at a payment date t *once* it fails to raise new equities to fulfill the required payment C_t^O. This occurs when its equity value immediately prior to the payment date (i.e., $E(t^-, V_{t^-})$) is lower than the repayment C_t^O.

- **(S.3)** Leland and Toft (1996) assume that the issuing firm would rollover matured bonds to keep the total amounts of outstanding bonds F and annual coupon payout C unchanged. The firm is also assumed to pay parts of its asset value V_t at a rate δ (i.e., $P_t = \delta V_t dt$ in Eq. (1)) to service bond repayments and dividend payout continuously. If the instantaneous payout $\delta V_t dt$ plus the gain of issuing new debt $(1-k)B(t, V_t)dt$[15] exceeds the bond repayment $(F + C)dt$, the remaining part goes to equity holders as dividend payout. Otherwise, the equity holders need to absorb the deficit $(F + C)dt - \delta V_t dt - (1-k)B(t, V_t)dt$. Note that $Fdt - (1-k)B(t, V_t)dt$ denotes the rollover losses (or gains if it is negative) for replacing a matured bond with an otherwise identical new bond. The firm files for bankruptcy at time t *once* equity holders fail to absorb the deficiencies; in other words, the equity value $E(t, V_t) = 0$.

(S.2) implicitly prevents an issuing firm from financing its required payments with its internal funds or debt capital, and this setting can protect the payments to short-term bond holders from harming the values of long-term bonds. **(S.2)** also increases equity holders' incentive to trigger default, because financing loan repayments with only equity capital would significantly dilute the value of original equity holders. That should be why adopting **(S.2)** would produce higher yield spreads for short-term bonds but generate downward-sloping term structures of yield spreads as illustrated by the dashed curves in **Fig. 1 (d)**.[16] In addition, unlike the Merton (1974)'s model that would significantly underestimate credit spreads for short-term bonds, adopting **(S.2)** could generate higher yield spreads for short-maturity bonds but have a greater possibility to overestimate the spreads of short-maturity junk bonds (see Eom et al., 2004). However, Eom et al. (2004) indicate that such kind of financing restrictions are rarely included in bond indentures. That could be why the downward-sloping yield spread curves generated by **(S.2)** do not fit the upward sloping nature (see **Fig. 1 (a)**) studied in Helwege and Turner (1999) and Huang and Zhang (2008).

Though **(S.3)** allows to finance debt repayments with internal funds and debt capital instead of solely equity capital as in **(S.2)**, its premise still makes structural models generate improper shapes of yield spread curves as illustrated by the dashed curves in **Fig. 1 (e)**. When the issuing firm is relatively unhealthy, equity holders needs to absorb the deficit $(F + C)dt - \delta V_t dt - (1-k)B(t, V_t)dt$

[15]The new bond should have the same covenants, like the coupon rate and the face value, as other outstanding bonds. This stationary debt setting is widely adopted in academic literature, such as Liu et al. (2006), Chen and Kou (2009), He and Xiong (2012), and etc. The issuance price $B(t, V_t)$ depends on the firm's financial status V_t and other market conditions at time t.

[16]Similar phenomena are also observed in Lando (2004).

due to very restrictive rollover and payout strategies. A poor financial status would significantly lower the price of new issuing bonds $B(t, V_t)$ due to the fixed coupon requirement, and tremendous rollover losses would cause equity holders to precipitate bankruptcy which may push bond yield spreads into an incredibly high level. Eom et al. (2004) report that the Leland and Toft (1996)'s model may overestimate bond yield spreads on average, and this can be verified by observing that the dashed curves in **Fig. 1 (e)** are higher than yield spread curves generated by other structural models. Eom et al. (2004) also claim that adopting **(S.3)** tends to overestimate the spreads of short-maturity bonds, which implies the downward-sloping term structures of yield spreads.

Contrary to the unified default boundary in **(S.1)**, our framework portrays the default trigger according to the payment schedule and covenants defined in the issuer's debt structure. Contrary to **(S.2)** and **(S.3)** that overly assume the firm's future refinancing policies, our framework preserves the nature of uncertainty about the future debt financing policies unless the strategies for coming due debts are likely to be certain as in Gopalan et al. (2014). Adopting **(S.1)**, **(S.2)**, and **(S.3)** may produce improper hump-shaped or downward-sloping yield spread curves as illustrated by the dashed curves in **Fig. 1 (c)**, **(d)**, and **(e)**, respectively. In contrast, the solid yield spread curves generated by our framework in **Fig. 1 (b)** can catch the upward-sloping nature found in empirical studies as exhibited in **Fig. 1 (a)**.

3 Our Quantitative Framework

In a structural model, all outstanding bonds and equities of the same issuer can be viewed as contingent claims on the issuer's asset value. Thus they can be evaluated by taking advantages of derivatives pricing methods. Our quantitative framework price these bonds/equities by enhancing the tree method since it is a flexible and popular pricing method that can easily deal with coexisting of multiple outstanding bonds as mentioned in Wang et al. (2014). In Section 3.1, we first show how the tree method models different financing assumptions and default triggers as discussed in Section 2.2. Next, we model the covenants that may change the payment schedule to analyze its impact on the values of outstanding bonds/equities and hence investors' decisions. Section 3.2 implements the payment blockage covenant that allows a previously issued senior bond to block the scheduled payments to newly issued junior bonds. The resulting framework allows us to explain why the order of loan repayment might not necessary be a key determinant for yield spreads (see Linn and Stock (2005)) as discussed in Section 4.2. To model early redemption of bonds due to exercising embedded call options of callable bonds, we develop a novel method, the forest, which is consisted of several trees to deal with the contingent changes of payment schedules due to premature redemptions as in Section 3.3. In Section 4.3, this framework is used to analyze the wealth transfer effect among different claim holders and resolve the conflicts of past empirical studies on call delay phenomena.

3.1 Tree Structures

Now we use a generic example illustrated in **Fig. 2** to demonstrate how a tree method can adjust its structure to simulate the asset value process of an issuer with multiple outstanding bonds under different financing assumptions and default triggers. To keep the illustrated tree structure and the following discussions simple, the firm is assumed to issue two bonds B_1 and B_2 with face value F_1 and F_2 and time to maturity T_1 and T_2, respectively.[17] The tree structure adjustments for modeling coupon

[17]More-than-two-bond case can be modeled by repeating the tree structure for simulating the payments and default triggers in **Fig. 2**.

payments are also ignored for simplicity. During the period without loan repayments, the evolution of the firm asset value reflects its investment risk and is mainly modeled by the well-known CRR binomial structure (denoted by solid lines) proposed by Cox et al. (1979). At a loan repayment date, says T_1, the "remaining asset" concept is implemented by subtracting the repayment $C^O_{T_1}$ (denoted by downward arrows) from the firm value $V_{T_1^-}$ (denoted by dashed circles) to obtain the remaining value $V_{T_1} \equiv V_{T_1^-} - C^O_{T_1}$ (denoted by boldfaced circles). The debt-structure-dependent default trigger is implemented by letting the firm default if its value is lower than the default boundary Θ_{T_1}, which is defined as the repayment $C^O_{T_1}$ plus the value of the frozen asset Λ_{T_1}. Specifically, the firm defaults and is liquidated if its asset value reach the nodes I, K, or L at time T_1 and no outgoing branches are emitted from these nodes. Note that this design captures the merit of the compound option approach; whether holders of B_2 can receive principal payments at time T_2 or not depends on whether the repayment at time T_1 is fulfilled or not. The trinomial structure (denoted by dashed lines) proposed by Dai and Lyuu (2010) is adopted here to adjust the tree structure. For example, the outgoing trinomial structure from the root node at time 0 can make one tree node, says I, coincide with the default boundary $\Theta(T_1)$ at time T_1 to avoid the nonlinearity error problem proposed by Figlewski and Gao (1999) from causing the tree to produce unstable pricing results. In addition, the outgoing trinomial structure from boldfaced nodes at time T_1 can avoid uncombined tree structure at the succeeding time step and decrease the computational cost for evaluating bonds/equities (see Dai and Lyuu (2010)). Note that this tree construction technique can also be used to model different financing assumptions and default triggers as discussed in Section 2.2. For example, the unified default boundary (i.e., **(S.1)**) assumption can be modeled by setting the default boundary Θ as the unified function defined in previous literature and by adjusting the tree to have a node that coincides with the boundary at every time step to avoid unstable pricing results. Rolling over bond B_1 at time T_1 by issuing another new bond B_3 with time to maturity T_3 and face value F_3 can be done by extending the tree structure from time T_2 to time T_3. The repayment at time T_3, $C^O_{T_3}$, is set as the repayments of principal F_3 and the coupon. The decrement of the firm value at time T_1 is set as $C^O_{T_1}$ minus $C^I_{T_1}$, where the latter part denotes the fund raised by issuing B_3.

3.2 Modeling Payment Blockage Covenant

Note that the credit quality of a loan repayment can be influenced by the scheduled repayments for other outstanding bonds of the same issuer as discussed in **Fig. 1**. However, the payment schedule may change due to enforcements of the payment blockage covenants — a covenant that grants the holders of a senior bond the right to block the payments to other junior bonds issued later than the senior one to ensure the payments for senior bond holders. This covenant has salient effect on determining the credit spreads as argued in Linn and Stock (2005) but is never quantitatively analyzed in any structural credit risk models, to our knowledge. Here we use the generic two-bond example in **Fig. 2** to demonstrate our analysis method. Let B_2 be a senior bond with this covenant to block the payments to the junior bond B_1. If the firms default at time t^*, then the payments to B_1's holders during the blockage period $[t^* - \eta, t^*]$ are blocked (if necessary) to satisfy the repayment of B_2, where η denotes the length of the blockage period determined in the covenant.

To model how a payment blockage covenant transfers the value from B_1's holders to B_2's holders, we define $\mathtt{BP}(t^*)$ as the value of "blocked payments"; that is, the value of total payments to B_1 holder occurred during the blockage period. We also define $\mathtt{UP}(t^*)$ as the value of the "unblocked payments"; that is, the value of total payments to B_1's holders during the time interval $[t^* - \eta - \Delta t, t^* - \eta)$, where

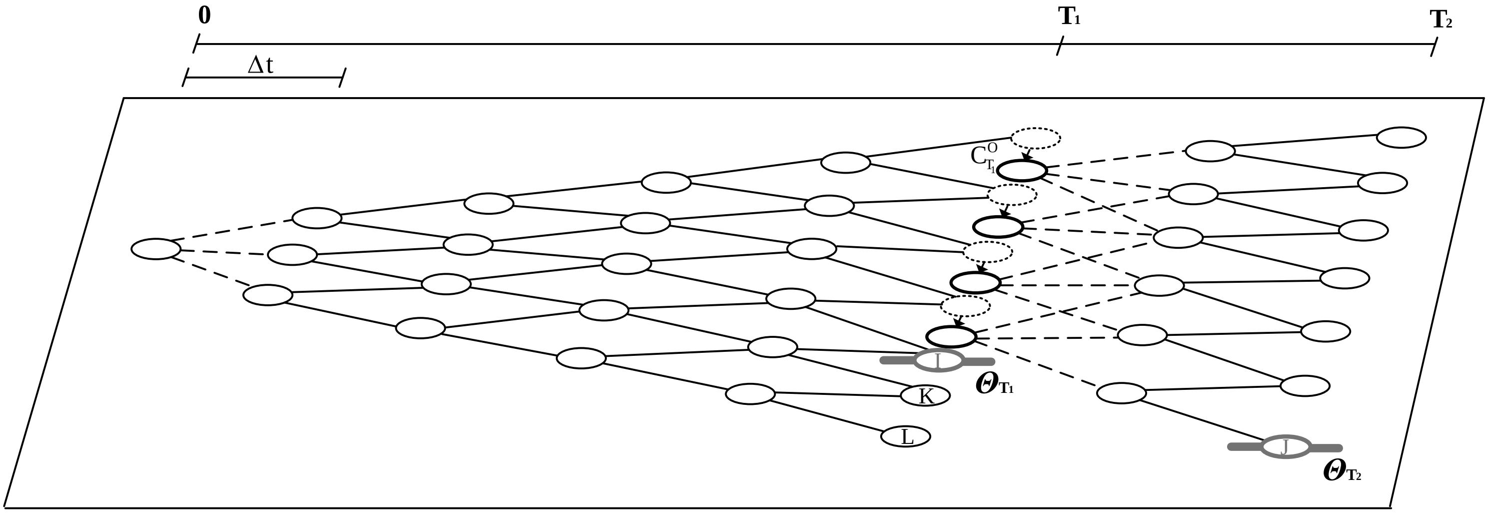


Figure 2: A Tree for Simulating the Dynamics of the Issuer's Asset Value. The firm is assumed to issue two bonds B_1 and B_2 matured at T_1 and T_2, respectively. Δt denotes the length of a time step. The CRR binomial structures proposed by Cox et al. (1979) are plotted by solid lines and the trinomial structure proposed by Dai and Lyuu (2010) are plotted by dashed lines. The boldfaced circles denote the remaining asset values after paying $C^O_{T_1}$ (marked by downward arrows) to redeeming B_1. The tree structure adjustments for modeling coupon payments are ignored for simplicity. Nodes I and J are decided to match the default boundaries (plotted by gray thick lines) to avoid unstable pricing results.

Δt denotes the length of a time step in the tree model. The payments occurred during this interval are no longer blocked and are guaranteed to be received by B_1's holders when we move from a node (in the tree) at time $t - \Delta t$ to the succeeding node at time t. The definitions of $\texttt{BP}(t^*)$ and $\texttt{UP}(t^*)$ can be used to clearly explain the real payments received by B_1's and B_2's holders under the potential enforcements of this covenant.

Since B_1, B_2, and equities E can be viewed as derivatives on the firm asset value, we can apply the risk-neutral valuation method to evaluate these contingent claims by summing the expected present values of the cash flows received by these claims. Here we use $E(t, V_t)$,$B_1(t, V_t)$, and $B_2(t, V_t)$ to denote the values of equities, B_1, and B_2 at time t with the remaining firm asset value V_t, respectively. The coupons C_1 and C_2 for bonds B_1 and B_2 are assumed to be paid semiannually. These three claims can be evaluated by applying the backward induction procedure on the tree illustrated in **Fig. 2** sketched as follows.

Case 1: At the long-term bond B_2 maturity date T_2.

$$E(T_2, V) = \begin{cases} V_{T_2} - F_2 - C_2/2 & \text{, if } V_{T_2} \geq \Theta_{T_2}, \\ 0 & \text{, if } V_{T_2} < \Theta_{T_2}, \end{cases}$$

$$B_2(T_2, V) = \begin{cases} F_2 + C_2/2 & \text{, if } V_{T_2} \geq \Theta_{T_2}, \\ \min(\texttt{BP}(T_2) + (1-\omega)V_{T_2}, F_2 + C_2/2) & \text{, if } V_{T_2} < \Theta_{T_2}, \end{cases}$$

and

$$B_1(T_2, V) = \begin{cases} \texttt{BP}(T_2) + \texttt{UP}(T_2) & , \text{if } V_{T_2} \geq \Theta_{T_2}, \\ \texttt{UP}(T_2) + \max(\texttt{BP}(T_2) + (1-\omega)V_{T_2} - F_2 - C_2/2, 0) & , \text{if } V_{T_2} < \Theta_{T_2}. \end{cases} \quad (3)$$

At time T_2, the firm survives if the firm value V_{T_2} is larger than the default boundary Θ_{T_2}; the residual of the firm value after repaying loans goes to equity holders. Simultaneously, the bond holders of B_2 receive the payments in full and the holders of B_1 receive all blockage payments $\texttt{BP}(T_2)$, the payments to B_1's holders occurred during the period $[T_2-\eta, T_2]$, plus the unblocked payment $\texttt{UP}(T_2)$, the payments to B_1's holders occurred during the period $[T_2-\eta-\Delta t, T_2-\eta]$. Note that these two values can be zero if B_1's maturity date T_1 is earlier than $T_2-\eta-\Delta t$. Otherwise, the firm defaults and the residual firm value (minus the liquidation cost) $(1-\omega)V_{T_2}$ plus the blocked payments $\texttt{BP}(T_2)$ is first used to satisfy the payments to B_2's holders $F_2 + C_2/2$. The remaining value (if any) plus the unblocked payment $\texttt{UP}(T_2)$ goes to holders of B_1.

Case 2: $0 \leq t < T_2$

At any time t prior to T_2, the firm defaults if its asset value V_t^- can not meet the default boundary Θ_t, defined as the repayment C_t^O plus the value of the assets frozen by restrictive covenants Λ_t. If the firm defaults, the firm is liquidated and equity holders receive nothing. The residual firm value (minus the liquidation cost) $(1-\omega)V_{t^-}$ plus the blocked payments $\texttt{BP}(T_2)$ is first used to satisfy B_2's required payments $\texttt{PV}_2(t)$, which is defined as the present value of all future unpaid payments to B_2's holders at time t. The remaining value (if any) plus the unblocked payment $\texttt{UP}(t)$ goes to holders of B_1.

On the other hand, the firm may be able to simultaneously fulfill the debt repayment C_t^O and satisfy the restrictive debt covenant. The remaining asset concept can be implemented by setting V_t as V_{t^-} minus C_t^O. The value of bond B_2 (or B_1) immediately prior to time t, denoted as $B_2(t^-, V_{t^-})$ ($B_1(t^-, V_{t^-})$), is equal to the payment received by holders of B_2 (B_1) at time t plus the continuation value — the expected present value of all B_2's (B_1)'s payments occurred after time t. The value of the former part is equal to zero at a non-repayment date. The value of the latter part, denoted as $B_2(t, V)$ ($B_1(t, V)$) for convenience, can be evaluated by the backward induction procedure that calculates the discounted expectation of the bond value at the next time step $B_2(t+\Delta t^-, V_{t+\Delta t^-})$ ($B_1(t+\Delta t^-, V_{t+\Delta t^-})$). Similarly, the equity value $E(t^-, V_{t^-})$ can be evaluated as the dividend paid at time t plus the continuation value $E(t, V_t)$ calculated by the backward induction. Thus we have

$$E(t^-, V_{t^-}) = \begin{cases} E(t, V_t) + \texttt{Dividend paid at time } t & , \text{if } V_{t^-} > \Theta_t, \\ 0 & , \text{if } V_{t^-} \leq \Theta_t \text{ and } C_t^O > 0, \end{cases} \quad (4)$$

$$B_2(t^-, V_{t^-}) = \begin{cases} B_2(t, V_t) + \texttt{coupon paid at time } t & , \text{if } V_{t^-} > \Theta_t, \\ \min(\texttt{BP}(t) + (1-\omega)V_{t^-}, \texttt{PV}_2(t)) & , \text{if } V_{t^-} \leq \Theta_t \text{ and } C_t^O > 0, \end{cases} \quad (5)$$

and

$$B_1(t^-, V_{t^-}) = \begin{cases} \texttt{UP}(t) + B_1(t, V_t) & , \text{if } V_{t^-} > \Theta_t, \\ \texttt{UP}(t) + \max(\texttt{BP}(t) + (1-\omega)V_{t^-} - \texttt{PV}_2(t^-), 0) & , \text{if } V_{t^-} \leq \Theta_t \text{ and } C_t^O > 0. \end{cases} \quad (6)$$

The coupon and the principal payments to B_1's holders can be blocked due to payment blockage covenant embedded in B_2. For example, if the firm defaults at time t, the present value of all payments to B_1's holders occurred within the period $[t-\eta, t]$ (denoted as $\texttt{BP}(t)$) can be blocked to satisfy the payments to B_2's holders $\texttt{PV}_2(t)$. The payment occurred within the period $[t-\eta-\Delta t, t-\eta)$ (denoted as $\texttt{UP}(t)$) just leaves the blockage period and is received by B_1's holders.

Linn and Stock (2005) empirically study whether the repayment order determined by the maturities of both senior and junior bonds would influence the credit quality of the senior bond. To analyze why their results contradict with the arguments of Ingersoll (1987) in Section 4.2, our framework can be slightly modified to analyze how holders of the short-term senior bond can block the payments to the long-term junior bond. Now let B_1 and B_2 be the short-term senior bond and the long-term junior one, respectively. The payment blockage covenant ceases to exist when the payments to B_1's holders are fully satisfied at time T_1. So a usual backward induction without sophisticated designs of "BP" and "UP" is applied to the time interval $[T_1, T_2]$. During the time interval $[0, T_1]$, the payment blockage covenant embedded in B_1 can block the payments to B_2, thus the procedure in aforementioned **Case 2** can be applied by swapping the roles of B_1 and B_2.

3.3 Dealing with Premature Redemptions with a Forest

Modeling premature redemptions by exercising call or put options embedded in bonds is still an open problem (see Jones et al. (1983)) due to complicated analyses of many possible debt structures and hence payment schedules caused by contingent bond redemptions. For example, calling back a long-term callable bond prior to the maturity of a short-term bond changes the order of principal payments and simultaneously transfer wealths among holders of equity, the short-term bond, and the long term one. To maximize the benefits of equity holders, the issuing firm optimizes its call decisions by comparing values of coexisting contingent claims under different debt structures. To evaluate contingent claims and analyze optimal redemption strategies, this section develops a novel quantitative framework, the forest, that are composed of several trees; each tree models one possible debt structure.

3.3.1 Forest Construction

Now we describe the structure of the forest for modeling a generic example that replaces the straight bond B_1 considered in **Fig. 2** with an otherwise identical callable bond. The resulting forest illustrated in **Fig. 3** is composed of two trees. The upper layer tree, like the tree in **Fig. 2**, models the dynamics of the firm's asset value under the condition that B_1 is not yet called. Thus the firm either repays $C^O_{T_1}$ at time T_1 (marked by downward arrows) to redeem B_1 or default once its asset value is lower than the default boundary Θ_{T_1}. The lower layer tree models the firm value dynamics under the premise that B_1 is called. Note that the firm does not require to redeem B_1 at time T_1 due to this premise. But it still needs to redeem B_2 at time T_2 and defaults once its value is lower than Θ_{T_2}.

For convenience, define $v(\phi)$ as the firm's asset value at a tree node ϕ. Calling the bond B_1 back would simultaneously change the firm asset value and the prevailing debt structure. For example, redeeming B_1 early at node U (located at time t in the upper layer tree in **Fig. 3**) would reduce the firm asset value $v(U)$ by the effective call price $\texttt{CP}$ and this change is denoted by a downward jump to node W. Note that B_1 is now removed from the debt structure and the outgoing trinomial branches from node W connect to its successor nodes, says X, Y, and Z, at time $t+\Delta t$ in the lower layer tree to reflect this removal. This trinomial structure is constructed by adopting Dai and Lyuu (2010)

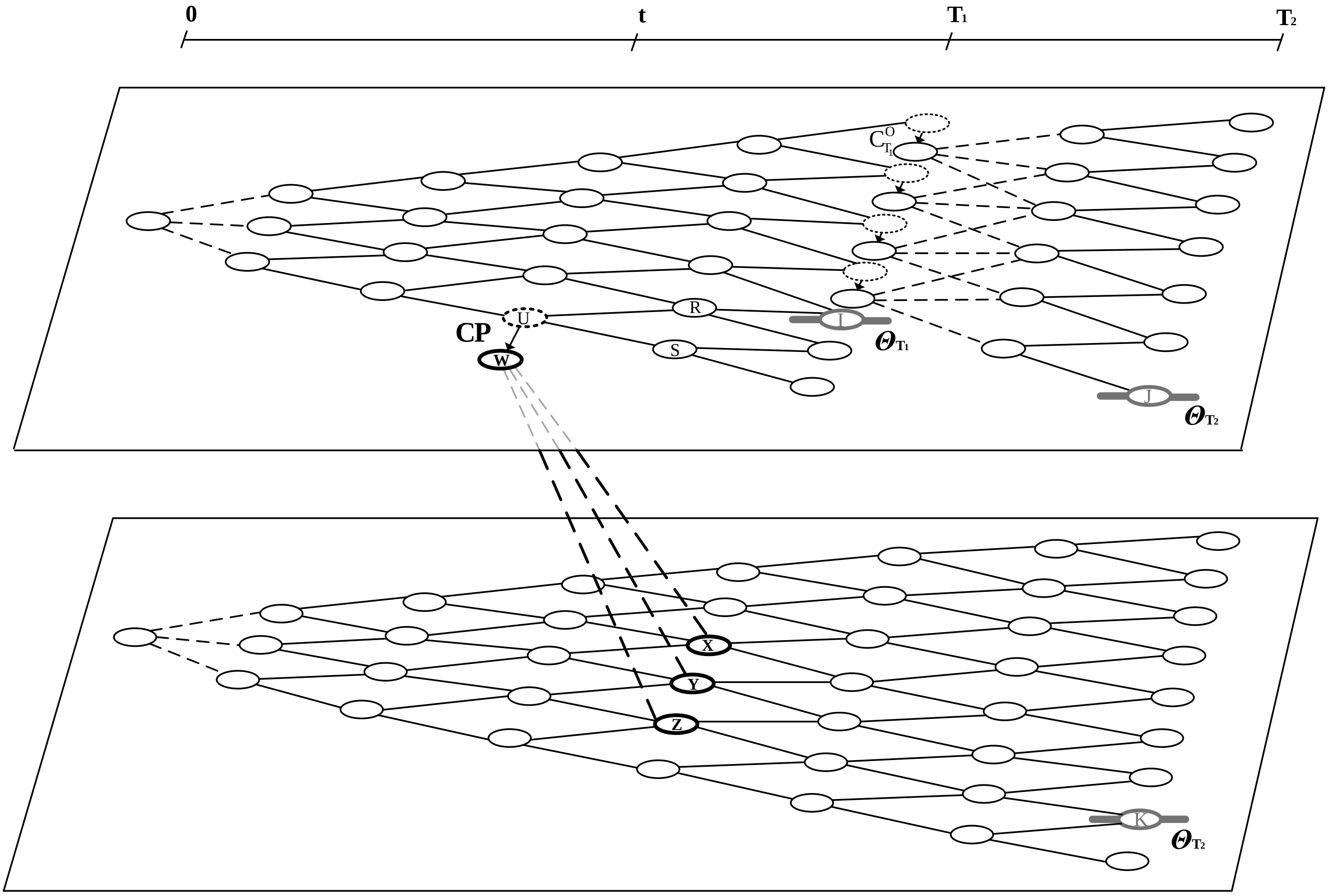


Figure 3: A Two-Layer Forest for Modeling a Callable Bond Early Redemption. All debt structure settings follow those in Fig. 2 except that B_1 is set as an otherwise identical callable bond. The evolution of the firm's asset value is modeled by the forest composed of two trees; the upper and the lower layer trees model the evolution of the firm's asset value given that B_1 is not called yet and B_1 is already called, respectively. The CRR binomial structure is plotted by the solid lines and the Dai and Lyuu (2010)'s trinomial structure is plotted by dashed lines. Nodes I, J, and K are decided to match the default boundaries to avoid unstable pricing results. Calling B_1 back would reduce the firm value by the effective call price `CP` (denoted by a downward jump to node W) and the prevailing debt structure (denoted by the trinomial structure connected to the lower layer tree.)

method that asymptotically simulates the firm value dynamics (see Eq. (1)) during the time interval $[t, t + \Delta t]$.

The above forest construction mechanism can be extended to deal with more complicated scenarios. For example, if a callable bond is also refundable, then the firm can call back the bond using the proceeds from issuing a new bond. Thus transiting from a upper layer tree (prior to early redemption) to a lower layer tree (after the redemption) would change the prevailing debt structure by replacing the callable bond with the new bond. The structure of the lower layer tree should be adjusted to reflect the change of the payment schedule as discussed in **Appendix A.1**. In addition, the forest composed of multiple trees can be constructed based on the aforementioned two-layer forest to model the debt structure with multiple outstanding callable bonds as in **Appendix A.2**. These trees can model the scenarios that one or some of callable bonds are redeemed early and resolve the intractable bond valuation problem raised in Jones et al. (1983). The resulting forest framework can analyze bond redemption strategies, the wealth transfer effect among claim holders, and phenomena as well as conflicts found in empirical studies in Section 4.3.

3.3.2 Decisions of Premature Redemptions

To maximize benefits of equity holders, a callable bond issuer determines whether it is optimal or not to redeem an outstanding callable bond back at each call date. Each tree in the forest simulates one possible debt structure caused by calling certain outstanding bonds back and can be used to evaluate the values of equity and bonds under this debt structure. Thus the issuer can compare equity values under different call strategies to find the best strategy.

Here we use node U at the aforementioned generic example illustrated in Fig. 3 to demonstrate the decision making process for calling B_1 back. Recall that the upper and the lower layer trees model the scenarios that B_1 is not yet called or is already called, respectively. If the issuer does not call B_1 back at node U, the "**n**on-called" equity value, denoted as $E^N(t^-, \upsilon(U))$, can be evaluated as the dividend paid at node U plus the continuation value. The continuation value denotes the time t's present value of future expected equity values at node U's successor nodes: R and S. These value can be evaluated by applying the backward induction. On the other hand, if B_1 is redeemed at node U, then the firm's asset value jumps downward from node U to node W to reflect the burden for repaying the effective call price `CP`. The outgoing branches from W connect to node X, Y, and Z located at the lower layer tree to reflect the removal of B_1 from the debt structure. Thus the "**c**alled" equity value, denoted as $E^C(t^-, \upsilon(U))$, can be evaluated as the dividend paid at node U plus the continuation value calculated by applying the backward induction on nodes X, Y, and Z.

Now the call decision can be made by comparing $E^C(t^-, \upsilon(U))$ and $E^N(t^-, \upsilon(U))$. If the former one is larger, the firm will redeem B_1 at node U, and the continuation value for B_2 is evaluated as the expected discounted B_2's values at nodes X, Y, and Z. If the latter one is larger, B_1 remains outstanding, and the continuation value for B_1 (B_2) are evaluated as the expected discounted B_1's (B_2's) values at nodes R and S. Note that the above call decision process can be applied to all the nodes located at called dates on the upper layer tree. The claims' values at a node that is not located at any call date can be evaluated by applying the backward induction on that node's successor nodes at the upper layer tree.

4 Numerical Results with Empirical Implications

By taking advantage of the flexibility of the tree method, our proposed quantitative framework can faithfully capture various aspects of a complicated debt structure, like the repayment schedule and the outstanding bonds' seniority, to match and explain past empirical studies. We resolve the numerically unstable problem addressed in Figlewski and Gao (1999) to produce stable pricing results as illustrated in **Appendix B**. We also propose a method to indirectly check the accuracy of our pricing results by taking the advantage of the capital structure irrelevance theory proposed by Modigliani and Miller (1958). The experiments in **Appendix C** illustrate how our framework quantitatively analyze the factors that influence the yield spreads of straight (or callable) bonds to explain the phenomena found in empirical studies, like Duffee (1998) and Avramov et al. (2007).

The repayment schedule may change contingently due to new issuances, trigger of bond covenants, or exercise of bond's embedded options. These changes would make evaluations of claim holders' values and analyses of their decisions difficult. This section will analyze four such scenarios by our quantitative framework discussed in **Section 3**. In **Section 4.1**, we analyze how the amounts and the maturities of newly issuing bonds dilute the values of remaining outstanding bonds to compare with the empirical results proposed by Collin-Dufresne et al.(2001b) and Flannery et al. (2012). The impacts of financing about-to-mature debt with internal or external funds under different levels of market liquidity on the yield spreads of remaining bonds are also studied to compare with He and Xiong (2012). In **Section 4.2**, we examine the impact of bond replacements and the payment blockage covenants on the payment priorities and hence the values of outstanding bonds to explain the studies of Linn and Stock (2005). The forest method proposed in **Section 3.3** can deal with callable/putable bonds. **Section 4.3** study how the call delay phenomenon is caused by the interaction effect (see Acharya and Carpenter, 2002) and the wealth transfer effect (see Longstaff and Tuckman, 1994). We then attempt to reconcile the conflicts between Longstaff and Tuckman (1994) and King and Mauer (2000) by finding the reasons, says the interest rate level during the sample period, that influence the empirical studies of redeeming callable bonds early. **Section 4.4** illustrates how a poison put can protect target firm's bonds against a leveraged buyout at the expense of the bidder as argued in Cook and Easterwood (1994) and Cremers et al. (2007).

In following experiments, all coupons are paid semiannually (i.e., $n = 2$ in Eq. (2)). We follow Welch (1997) and Gorton and Kahn (2000) to let bank loans be the most senior bond type in a corporate debt structure. All bonds are assumed to be debentures except that bank loans are fully secured by the firm's asset. In addition, all newly issued bonds are assumed to be issued at par for consistency. Numerical settings, such as the interest rate r, the firm's asset volatility σ, the tax rate τ, the bankruptcy cost ω, and the bond issuance cost k are basically follow Leland (1994) and He and Xiong (2012).

4.1 Analyzing Planned Issuances and Rollovers

4.1.1 Issuance Strategies and the Claim Dilution Effects

In addition to analyzing a complicated debt structure, our framework can also analyze the impacts of issuing new bonds or rolling over matured bonds on the values of other unmatured bonds and equities. This allows us to explore the factors that influence yield spreads and hence analyze bond issuance strategies. Note that a planned issuance now (or in the near future) would increase the current (or the future) leverage ratio of the issuing firm and hence increase future required payments, the default likelihood, as well as bonds' yield spreads. This might partially explain why Collin-Dufresne and Goldstein (2001a) argue that the bond yield spreads contain the information of the firm's current leverage and investors' expectations about the firm's future leverage. In addition, Flannery et al. (2012) find that the impact of the expected future leverage on yield spreads is much more salient than the current leverage. To analyze the influence of current/future leverage on yield spreads, we can introduce planned issuances of bonds to change the leverage ratio and analyze the corresponding impacts on yield spreads as illustrated in **Fig. 4**. The leverage ratio at time t, $\mathtt{LEV}_t$, is calculated by following the definition of Collin-Dufresne et al. (2001b) as

$$\mathtt{LEV}_t = \frac{(\mathtt{Total\ Face\ Value\ of\ Bonds})_t}{(\mathtt{Total\ Face\ Value\ of\ Bonds})_t + (\mathtt{Market\ Value\ of\ Equity})_t},$$

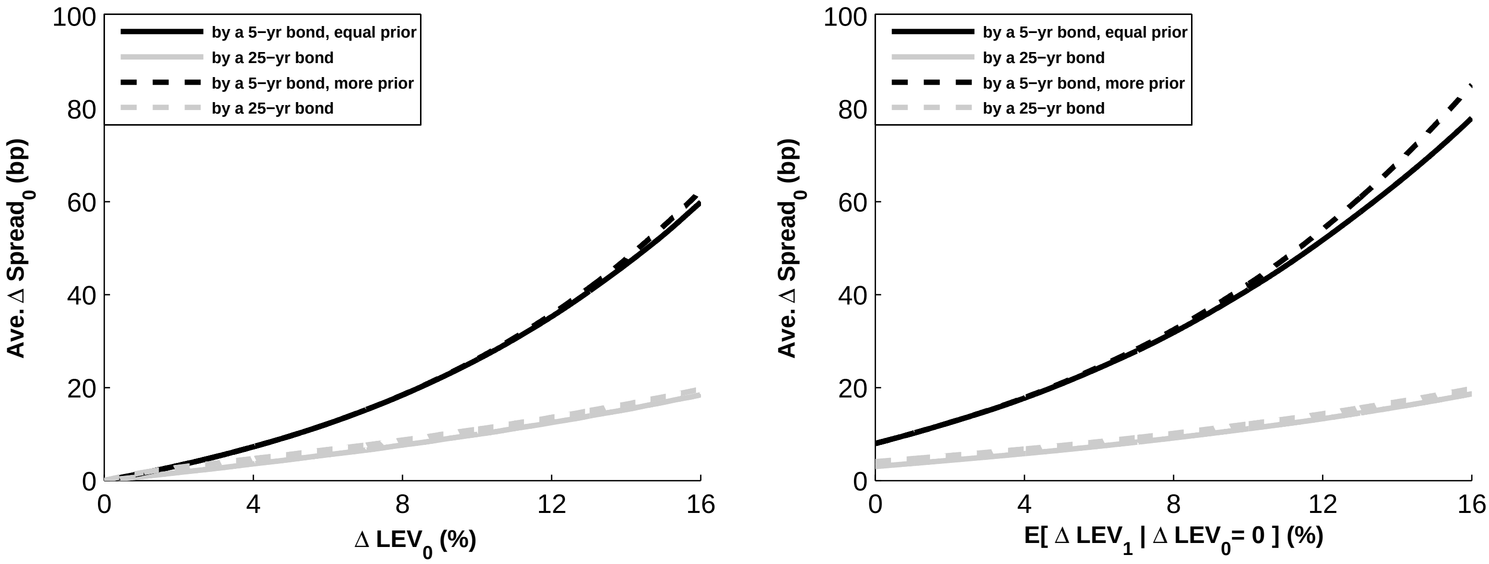
where $(\texttt{Market Value of Equity})_t$ is evaluated by our framework. The future leverage in panel (b) is proxied by expected future leverage one year ahead as in Flannery et al. (2012). It can be observed that the change of leverage ratio is positively related to the change of yield of outstanding bonds' portfolio regardless the issuance date, seniority, and the maturity date of the newly issuing bond. By comparing the curves in panel (b) with those in panel (a), our quantitative framework confirms Flannery et al. (2012)'s finding that the increment in the future leverage ratio has greater impact on the change of yield of the portfolio than the increment in the current leverage ratio.

Note that our quantitative framework can also analyze the impacts of the factors other than the leverage ratio. By comparing black curves with gray ones, we can observe that the short-term (5-year) new issuance has more significant claim dilution effect on other outstanding bonds than the long-term (25-year) one. This observation is consistent with the argument of Ingersoll (1987) that bond repayments may deteriorate the credit quality of unmatured bonds. Similarly, by comparing solid curves with dashed ones, we can observe that the credit quality of other outstanding bonds deteriorates with the improvement of the repayment priority of the new issuance. Note that the impacts of the maturity or the seniority of the newly issued bond become more significant with the increment of the bond issue size (or the leverage ratio). Besides, the rollover risk can also be estimated by our framework as in panel (b). Specifically, the proceeds of the new issuance occurred at one year later would be firstly used to finance the bond matured at year 1 (B_1) and thus the increment in the yield of portfolio given that the future expected leverage ratio is zero purely reflects the risk for rolling over B_1. A more detail analysis for rollover risk will be discussed in the next subsection.

4.1.2 The Valuation Effect of Rolling Over a Maturing Bond on the Existing Bonds Considering Rollover Risk via the Channel of Bond Market Illiquidity

The impact of the potential rollover risk implied by the issuer's debt structure on its creditworthiness is widely studied recently. Gopalan et al. (2014) empirically confirm that an issuer with a greater proportion of bonds that will be rolled over within one year are more likely to experience severe credit deteriorations; therefore, long-term bonds of the same issuer are more likely traded at higher yield spreads. This phenomenon is found to be more significant during recession years. Nagler (2014) study how the illiquidity of bonds raises the rollover risk and hence the entire term structure of bond yield spreads.

To examine the negative impact for repaying the short term bond on other outstanding bonds of the same issuer, **Fig. 5 (a)** displays that the yield spread of the 16-year bond (denoted by the y-axis) increases with the increment of the 1-year bond's face value given that the issuer's leverage ratio (or the total amount of outstanding bonds) unchanged. Note that the dotted curves denote the scenario that the 1-year bond is financed by internal funds and the differences between solid (or dashed) curves and dotted ones reflect the cost of issuing new bonds and the rollover risk. Obviously, the differences of yield spreads increase with the increment of the 1-year bond's face value, which are consistent with the aforementioned Gopalan et al. (2014)'s observation that credit deteriorates with the increment of the proportion of short-term bonds being rolled over. Note that the issuer's financial status (proxied by the initial firm asset value V_0) is negatively related to the illiquidity of bonds and hence the issuance cost k (defined in Section 2.2). Here we compare the roll over risk between a good financial status (with a high V_0 1000 and a low k 1%) plotted in black curves and a poor status (with a low V_0 700 and a high k 2%) plotted in gray curves. The yield spread difference for poor financial status tends to be more significant than the difference for good status; this reveals an asymmetric effect of rollover

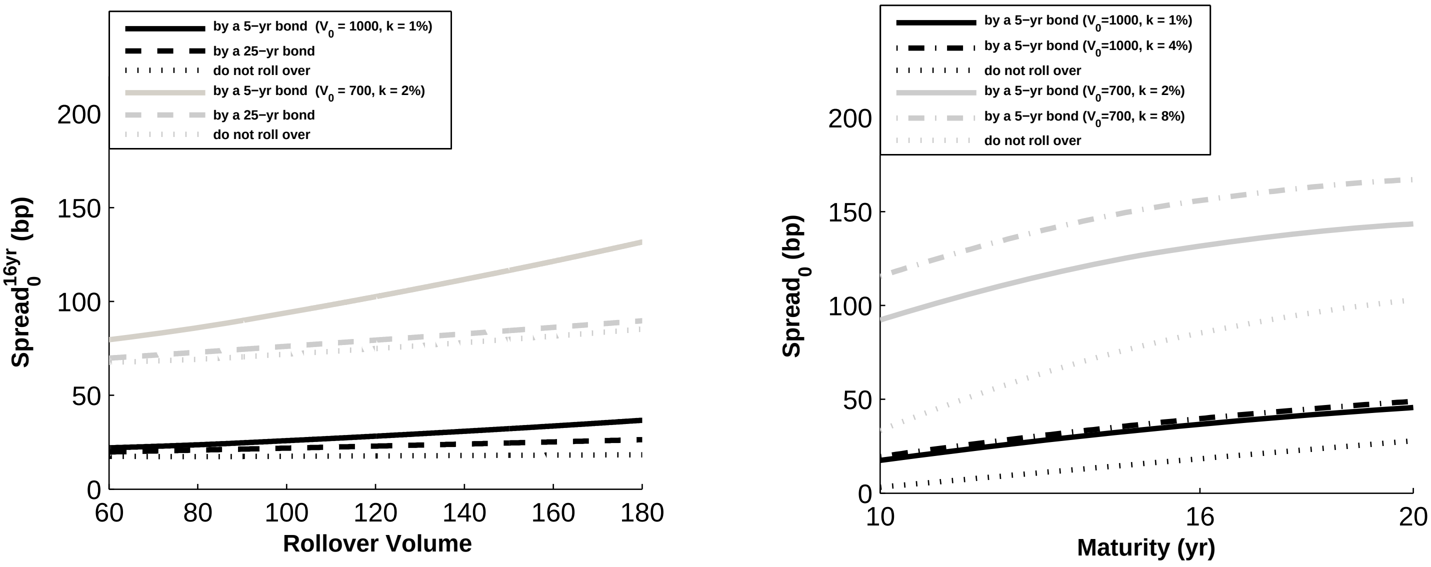


(a) (b)

Figure 4: The Impact of Issuances on the Yield of Portfolio of Outstanding Bonds. The issuer is assumed to have five equal-priority \$100 outstanding bonds with remaining time to maturities 1, 10, 16, 20 and 30 years and coupon rates 7%, 8%, 10%, 11% and 14%, respectively. The initial firm's asset value V_0 is 1000, its volatility $\sigma = 20\%$, the risk-free rate $r = 6\%$, the issuance cost $k = 1\%$, the liquidation cost $\omega = 50\%$, and the tax benefit $\tau = 35\%$. The x and the y axes denote the change of leverage ratio and the change of the yield of the portfolio of outstanding bonds due to new issuances, respectively. Panel (a) and (b) denote the new issuance occurred immediately or in one year later, respectively. The $\Delta\texttt{LEV}_t$ in the x-axis represents the change of leverage ratio at time t due to a new issuance. The black and gray curves denote the maturity of newly issued bonds are 5 and 25 years, respectively. The solid and dashed curves denote the seniority of the new bond is equal to or is more senior to other outstanding bonds.

risk on credit risk: rollover risk is much more devastating to unhealthy firms than to healthy firms. Besides, the rollover risk is also influenced by the maturities of bonds issued to finance repayments. Here we compare the roll over risk between a long-term 25-year bond issuance plotted in dashed curves with a short term 5-year bond issuance plotted in solid curves. The yield spread difference for the short-term bond issuance tends to be more significant; and this reveals another asymmetric effect: rollover risk is much more salient as the maturing bond is replaced by a short-term bond issuance. By combining these two asymmetric effects, we can explain Kahl et al. (2015)'s finding that short-term bonds are less likely to be used by relatively unhealthy firms to replace maturing bonds to alleviate the negative impact of rollover risk on creditworthiness.

Fig. 5 (b) plots yield spread curves under different financing strategies and issuer's financial statuses. It can be observed that rollover risks rise entire yield spread term structures and that the aforementioned two asymmetric effects are preserved for different maturities. Moreover, issuance costs would increase further in recession bond markets which would increase bond yield spreads further (by comparing solid curves with dot-dashed curves).



(a) (b)

Figure 5: Rollover Risk Analyses. The debt structure of the issuer is the same as the five-outstanding-bond structure studied in **Fig. 4** except that the face values of the 1-year bond in panel (a) and (b) are denoted by x-axis and 180, respectively. The face value of the 30-year bond is tuned to make the total amount of outstanding bonds equal to 500. In both panels, the black and gray colors represent the good financial status case ($V_0 = 1000$ with a low issuance cost $k = 1\%$ or 4%) or the poor one ($V_0 = 700$ with a high cost), respectively. Dot curves denote the scenario that the 1-year bond is financed by the issuer's internal fund. In panel (a), the y-axis denotes the yield spread of the 16-year bond. At the maturity of the 1-year bond, the repayment will be financed by issuing a new 5-year bond (denoted by solid curves), or by a new 25-year bond (denoted by dashed curves). Panel (b) displays yield spread curves implied by coexisting 10-, 16- and 20-year bonds given that the 1-year bond is planed to be rolled over by another 5-year bond. The solid curves and dash-dotted curves denote the low issuance and the high issuance cost due to different market liquidity conditions. Other numerical settings are $r = 6\%$, $\sigma = 20\%$, $\tau = 35\%$ and $\omega = 50\%$.

4.2 Bond Replacements and Payment Blockage Covenants

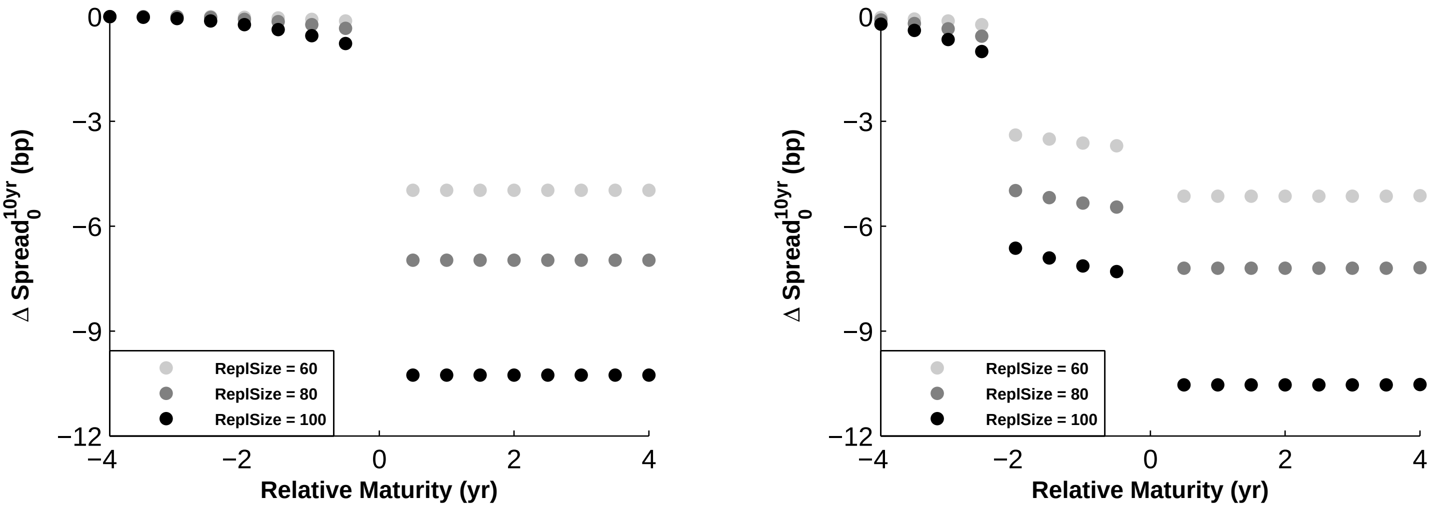
The yield spread of a bond depends on its effective repayment priority determined by its seniority and the payment schedule of the issuer. For example, the repayment of a short-term junior bond may weaken the issuer's financial status that deteriorates the effective repayment priority of another long-term senior bond. This effective priority may change due to replacements of other coexisting bonds or existences of payment blockage covenants. Our framework can measure the impacts of changing effective priorities on bond yield spreads to explain phenomena found in empirical studies. Linn and Stock (2005) show that replacing an existing bank loan (`BL`) with a new junior bond (`NewJB`) would decrease the yield spread of another outstanding senior bond (`SB`), and the decrement magnitude increases with the level of the replacement size. Apparently, this is because replacing the most senior `BL`[18] with a `NewJB` would improve the relative priority of `SB`. Surprisingly, whether `NewJB` will mature before or after the maturity of the `SB` does not affect the decrement magnitude of `SB` yield spread significantly unless the issuer's creditworthiness deteriorates further. This observation contradicts

[18] Welch (1997) and Gorton and Kahn (2000) suggest that a bank loan is usually the most senior debt in a corporate debt structure.

Ingersoll (1987)'s inference that repayments to $\texttt{NewJB}$ prior to the maturity of $\texttt{SB}$ should have salient dilution effect on $\texttt{SB}$. One possible reason is that the previously issued $\texttt{SB}$ is protected by the payment blockage covenant which grants $\texttt{SB}$ holders the right to block the scheduled payments to $\texttt{NewJB}$ holders occurred during the so-called payment blockage period to assure that the repayments to $\texttt{SB}$ are fulfilled. Our framework can model this covenant as discussed in **Section 3.2** to analyze the aforementioned empirical findings and conflicts as follows.

To examine the impact of debt replacement and the payment blockage covenant, we study a hypothetical five-bond debt structure modified from the one considered in the last section to fit the scenario analyzed by Linn and Stock (2005) as illustrated in **Fig. 6**. Note that the $\texttt{BL}$ is replaced by a new $\texttt{BB}$ with the same face value and the maturity to make our analysis only focus on the impact of changing the relative priority due to the bond replacement. It can be observed from both panels that the bond replacement would decrease the yield spread of $\texttt{SB}$ (denoted by negative $\Delta\texttt{Spread}_0^{\text{10yr}}$), and this phenomenon is more significant with the increment of the bond replacement size. These results are consistent with the empirical studies of Linn and Stock (2005). To analyze the conflict between Ingersoll (1987) and Linn and Stock (2005) on the impact of the relative maturity between $\texttt{NewJB}$ and $\texttt{SB}$, we compare the influence of the absence or the existence of the payment blockage covenant with a 2-year blockage period in panel (a) and (b), respectively. In panel (a), whether the $\texttt{NewJB}$ will mature before or after the $\texttt{SB}$ (denoted by negative or positive relative maturity) significantly influence the magnitude of $\Delta\texttt{Spread}_0^{\text{10yr}}$, which is consistent with the aforementioned Ingersoll (1987)'s argument. However, introducing the payment blockage covenant as illustrated in panel (b) can protect the benefits of $\texttt{SB}$ holders by blocking the payments to $\texttt{NewJB}$ occurred within the blockage period. Therefore, given that $\texttt{NewJB}$ matures prior to the maturity of $\texttt{SB}$ but within the blockage period (i.e, the relative maturity is within the range $[-2, 0)$), the decrement of $\Delta\texttt{Spread}_0^{\text{10yr}}$ in panel (b) is more significant than those illustrated in panel (a). This could explain why Linn and Stock (2005) find that the relative maturity is not a significant factor for explaining $\Delta\texttt{Spread}_0^{\text{10yr}}$.

Linn and Stock (2005) also show that the issuing firm's financial status could influence the explanatory power of the relative maturity on $\Delta\texttt{Spread}_0^{\text{10yr}}$, and we analyze this by using different firm values V_0 to proxy different statuses as in **Fig. 7**. In panel (a), the decrement of $\Delta\texttt{Spread}_0^{\text{10yr}}$ becomes more significant with the decrement of V_0, which denotes the credit enhancement of $\texttt{SB}$ provided by the debt replacement becomes more significant with the deterioration of the issuer's financial status. To imitate the analyses of regression that estimates relationships among different factors by aggregating the impacts of these factors' values from collected samples, we calculate the average of $\Delta\texttt{Spread}_0^{\text{10yr}}$, denoted as $\texttt{Ave.}\Delta\texttt{Spread}_0^{\text{10yr}}$, under different scenarios in panel (b). For example, the node A at the solid black curve denotes the scenario that the $\texttt{NewJB}$ matures later than the $\texttt{SB}$, V_0 is 800, and the $\texttt{SB}$ contains the payment blockage covenant. Its value is calculated by averaging the values of eight sample points in the dashed circle in panel (a). This allows us to measure the average impact of V_0 on the $\Delta\texttt{Spread}_0^{\text{10yr}}$ under different scenarios listed in the bottom right corner in panel (b). Given that the payment blockage covenant is present (dented by solid curves), the difference between the scenario that the relative maturity is negative (denoted by the gray solid curve) and positive (the black solid curve) becomes more significant with the decrement of V_0; this confirms Linn and Stock (2005)'s finding that the explanatory power of the relative maturity becomes more significant when the issuer's financial status deteriorates. Besides, the credit enhancement effect provided by the payment blockage covenant can be measured by the differences between dashed curves and solid curves. It can be observed that this effect is insignificant when $\texttt{NewJB}$ matures later than the maturity of $\texttt{SB}$ by comparing black dashed and solid curves. On the other hand, by comparing gray dashed and solid curves, this effect becomes more significant with the deterioration of the issuer's financial status when `NewJB` matures earlier than the `SB`'s maturity.

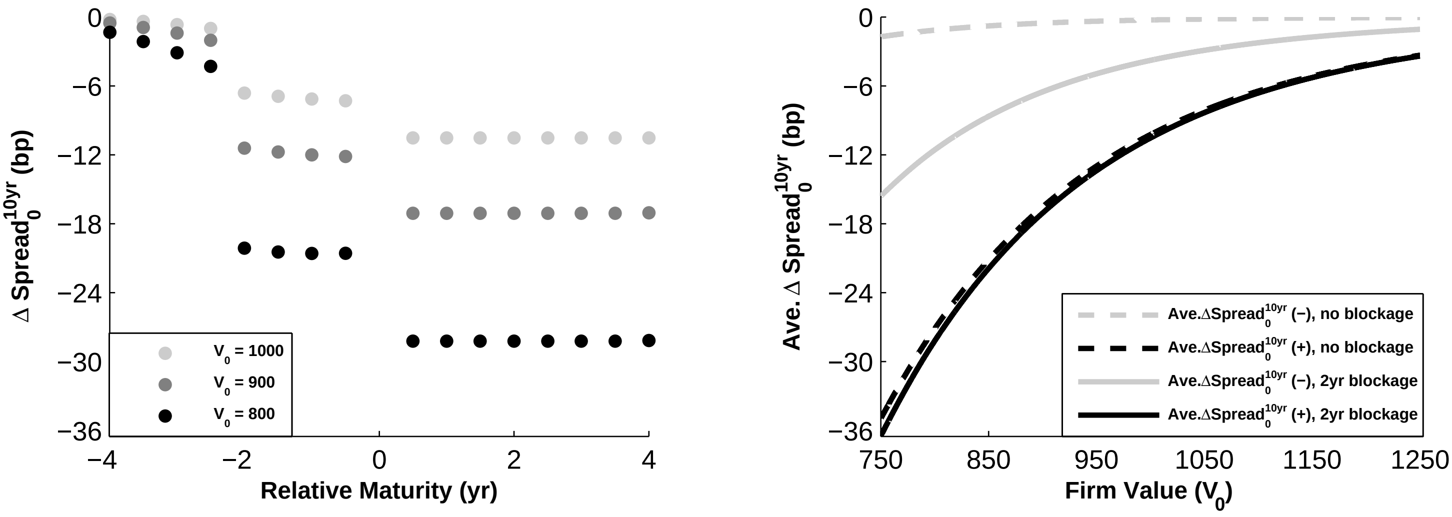


(a) (b)

Figure 6: The Impact of the Bond Replacement and the Payment Blockage Covenant on the `SB`'s Yield Spread Change. The debt structure is the same as the five-bond structure studied in **Fig. 4** except that the 1-year bond is changed to a bank loan (`BL`), the 10-year bond is set as a senior bond (`SB`), and other bonds are set as junior ones. For comparisons, three different face values of `BL` (or the replacement size "`ReplSize`" by issuing `NewJB`) are denoted by different colors illustrated in the lower left corners in both panels. The face value of the 30-year bond is tuned to make the total amount of outstanding bonds equal to 500. The "`Relative Maturity`" on the $x-$axis is defined as the maturity of `NewJB` minus the maturity of `SB`. The $\Delta\texttt{Spread}_0^{10\texttt{yr}}$ on the $y-$axis denotes the change of the senior bond yield spread by replacing `BL` with `NewJB`. Panels (a) and (b) consider the absence or existence of the payment blockage covenant with a 2-year blockage period, respectively. Other numerical settings are $V_0 = 1000$, $r = 6\%$, $\sigma = 20\%$, $k = 1\%$, $\tau = 35\%$, and $\omega = 50\%$.

4.3 Optimal Call Policies for Complex Debt Structures

The forest method introduced in Section 3.3 can model the contingent change of the issuer's capital structure due to exercise of options embedded in outstanding bonds. Thus we can theoretically analyze claim holders' exercise decisions to explain phenomena or conflicts found in past empirical studies. This section analyzes call policies for American-style callable bonds that can be redeemed by the issuer at any time prior to maturity. Brennan and Schwartz (1977) and Ingersoll (1997a) suggest that a callable bond should be redeemed immediately once its market value exceeds the effective call price (i.e., the call price plus the accrued interest). This strategy, called as the "textbook policy" by Longstaff and Tuckman (1994), can explain the empirical phenomenon that a low interest rate environment entails high bonds' values and a high likelihood for early redemptions. However, many empirical studies find that the market value of a about-to-call bond is usually higher than its effective call price, which entails that the issuer tends to defer the call decision. In addition to market frictions like the tax mentioned in Ingersoll (1977b), the call delay phenomena might result from the interaction effect and the wealth transfer effect. The former effect proposed by Acharya and Carpenter (2002)



(a) (b)

Figure 7: The Issuer's Financial Status, the Relative Maturity, and the Significance of the Payment Blockage Covenant. The debt structure is identical to the structure studied in Fig. 6 except that the face value of `BL` (or the bond replacement size) is set as 100. Panel (a) displays the impact of the bond replacement on the yield spread change of the `SB` with a 2-year blockage covenant under different financial status (proxied by different V_0 listed in the lower left corner). Panel (b) illustrates the impacts of the issuing firm financial status and the payment blockage covenant on the average yield spread change of `SB` (denoted by $\texttt{Ave.}\Delta\texttt{Spread}_0^{10\text{yr}}$ in y-axis). In the bottom right corner, "−" and "+" denote the range of the relative maturity are within the range $[-4, 0)$ and $(0, 4]$, respectively. Solid curves and dashed ones indicate that the `SB` contains a payment blockage covenant or not, respectively. Other numerical settings are $r = 6\%$, $\sigma = 20\%$, $\tau = 35\%$ and $\omega = 50\%$.

states that the issuer tends to defer call decisions since an immediate redemption decision would also ruin the issuer's option to potentially default on callable bonds in the future. The latter effect, studied by Jones et al. (1984) and Longstaff and Tuckman(1994; hereafter LT), denotes that, under a complex debt structure with multiple outstanding bonds, an early redemption of one bond may redistribute wealth to holders of other outstanding bonds other than equity holders. This effect would defer the issuer's call decision to protect the interests of equity holders. However, the empirical studies of King and Mauer (2000; hereafter KM) empirically reject LT's argument. The following experiments will first quantitatively examine these two effects and then explore the reason of inconsistent empirical results made by LT and KM.

To capture the impacts of the interaction and the wealth transfer effects on call decisions, **Table 1** displays the evaluation results for different bonds holders and equity ones under hypothetical five-bond debt structures. All outstanding bonds are straight bonds except that B_3 is a callable bond under $\mathcal{D}_c^{\mathrm{T}}$ and $\mathcal{D}_c^{\mathrm{M}}$. The superscripts "T" and "M" denote that the call policy for B_3 is the **t**extbook policy and is the policy to **m**aximize the equity value, respectively. By comparing the values of bonds and equity in the case $\mathcal{D}_c^{\mathrm{T}}$ with those in the otherwise identical five-straight-bond case $\mathcal{D}_s$, the potential early redemption of B_3 under the textbook policy significantly decreases B_3's value and corresponding benefits are redistributed to holders of B_4 and B_5 matured later than the maturity of B_3. This is because the burden for repaying the principal of B_3 would mainly deteriorate the credit quality of B_4 and B_5 rather than B_1 and B_2; thus the benefit for redeeming B_3 early would mainly compensate the holders of B_4 and B_5. The textbook policy might even harm the benefit of equity holders since the early redemption benefits are mainly absorbed by the holders of B_4 and B_5. Obviously, the textbook policy is suboptimal to equity holders when there are multiple outstanding bonds in the debt structure.[19]

The call policy to maximize equity holders' benefits seems to be more reasonable and can simultaneously explain the call delay phenomenon and the interaction effect. By comparing the values of bonds and equity in the case $\mathcal{D}_c^M$ with those in $\mathcal{D}_c^T$, the values of the callable bond B_3 in the former case are larger than those in the latter one. This value increment seems to be because the issuer tends to delay the call decision until the market value of B_3 becomes much higher than the effective call price. This call policy also decreases the values of B_4 and B_5 and increase the value of E, which reflects that it can alleviate the wealth transfer effect (compared to the textbook policy) to improve the benefit of equity holders. Besides, by comparing the values of B_3 in the case $\mathcal{D}_c^M$, the interaction effect can be captured by observing that B_3 appreciates when V_0 slightly decreases. This is because the issuer tends to delay the call decision in a poor financial status to avoid ruining its another option to default on callable bonds in the future.[20]

V_0		1000			1250			1500		
		$\mathcal{D}_s$	$\mathcal{D}_c^T$	$\mathcal{D}_c^M$	$\mathcal{D}_s$	$\mathcal{D}_c^T$	$\mathcal{D}_c^M$	$\mathcal{D}_s$	$\mathcal{D}_c^T$	$\mathcal{D}_c^M$
B_1	(5-yr)	103.87	103.87	103.87	103.87	103.87	103.87	103.87	103.87	103.87
B_2	(10-yr)	113.23	113.23	113.23	113.90	113.90	113.90	114.07	114.07	114.07
B_3	(16-yr)	134.67	**101.90**	**125.36**	137.59	**101.90**	**122.65**	138.72	**101.90**	**117.16**
B_4	(20-yr)	148.04	150.95	148.09	152.40	154.25	152.49	154.34	155.44	154.47
B_5	(30-yr)	194.18	198.36	194.32	201.13	204.00	201.36	204.46	206.58	204.76
E		477.20	469.96	477.48	715.82	712.83	716.34	960.81	960.13	961.61

Table 1: The Wealth Transfer and the Interaction Effects. The debt structure is identical to the structure studied in Fig. 4 except that B_3 is set as a callable but non-refundable bond with a call price of \$100 under the cases $\mathcal{D}_c^T$ or $\mathcal{D}_c^M$. Specifically, $\mathcal{D}_s$ denotes the case that all outstanding bonds are straight ones. $\mathcal{D}_c^T$ and $\mathcal{D}_c^M$ denote the cases that the callable bond B_3 are redeemed under the textbook policy and under the policy that maximizes the equity holders' benefits, respectively. The numerical settings are the same as those in Fig. 4 except V_0 defined in the first row. All outstanding bonds values and the equity value E for each scenario are listed under that scenario.

To measure the impact of only the interaction effect on the call delay, we consider a simple debt structure containing only 1 callable bond B'. The impact of the wealth transfer effect can be measured by comparing the one bond debt structure with the five-bond structure. The latter structure is designed to contain 3 straight bonds and 2 callable bonds with time to maturity at year 10 (short-term callable bond B_2) and at year 16 (long-term bond B_3) to analyze call strategies for multiple callable bonds empirically studied in KM. Recall that under the textbook policy, a callable bond is redeemed once its market value exceeds the accrued interest plus the call price, which is set to the face

[19]Actually, LT state that the textbook policy is optimal only if the issuer's capital structure does not change during the early redemption process. That is, "the bond has to be refunded with an issue that has exactly the same remaining interest payments, sinking fund provisions, and option features as the original issue." However, "many callable bonds are not refundable."

[20]Jacoby and Shiller (2010) also empirically confirm that the presence of the issuer's default risk indeed affect its call policy.

value of the callable bond in our experiment. This entails that a callable bond should be redeemed once the interest rate r is lower than its coupon rate. However, all call boundaries implied by the equity-value-maximization policy illustrated in **Fig. 8(a)** denote that the issuer tends to defer the call decision until r reaches a much lower level. For example, the call delay phenomenon of B' can be illustrated by the difference between the call boundary of B' plotted in the dashed curve and the call boundary of the textbook policy[21] r=10% (the coupon rate of B'). This difference also purely reflects the impact of the interaction effect on the call delay phenomenon. In addition, the difference (call delay phenomenon) is more pronounced with the decrement of the prevailing firm asset value V_5 due to the issuer's concern of the value of the default option that increases with the decrement of V_5.

The impact of the wealth transfer effect on the call delay phenomenon can be illustrated by the difference between the one-bond debt structure's call boundary (denoted by the dashed curve) and the five-bond structure's ones (denoted by solid curves). It can be observed that the issuer with multiple outstanding bonds tends to defer the call decision until r reaches a much lower level. Similarly, KM observe that the long-term callable bond is prone to be redeemed early than the short-term one and this can be confirmed by comparing the call boundaries for B_2 (denoted by the gray solid curve) and for B_3 (denoted by the black solid one). Note that both boundaries overlap when the interest rate level r is low, which entails that both bonds are simultaneously redeemed.

KM empirically reject the relation between the wealth transfer effect and the call delay phenomenon by introducing a new measure: premium over effective call price (abbreviated as `PoCP`). It is defined as the market value for a about-to-call bond (or the bond value at a selected time point if it is never redeemed early) minus the effective call price. We can standardize `PoCP` (abbreviated as `SPoCP`) by dividing it by the call price and use it to analyze the call delay phenomenon as KM do in **Fig. 8 (b)**. KM argue that the relation between `SPoCP` and V_0 should form a hump-shaped curve if the wealth transfer effect were a key determinant; this is because a callable bond might appreciate with a slight deterioration in the firm's creditworthiness[22] due to the issuer's call delay decisions to alleviate the wealth transfer effect. Note that this argument can also explain that the interaction effect would cause the hump-shaped relation as illustrated by the dash curve. In addition, the wealth transfer effect strengthens this relation as illustrated by the solid curve. However, KM empirically show that the relation between `SPoCP` and V_0 is significantly monotonically increasing and the hump-shaped relation is insignificant *on average*.

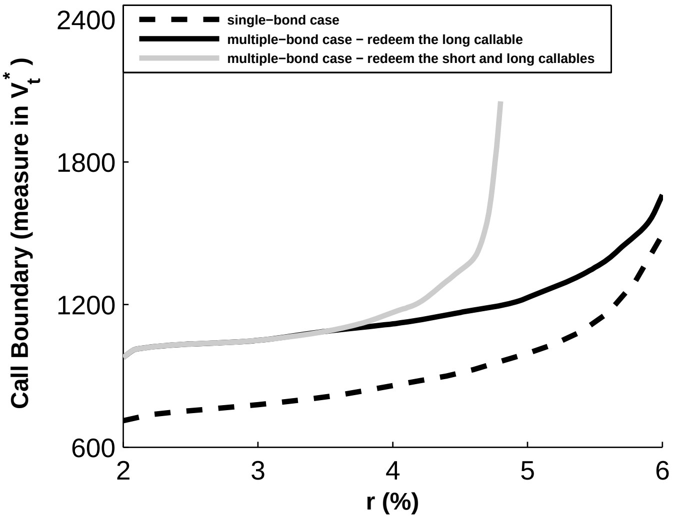
The conflict between LT and KM might be because the proportion of callable bonds that are not redeemed early in KM's sample set is much larger than the proportion in LT's one. Since the characteristics of callable bonds that are not redeemed early are similar to those of straight bonds, KM's empirical studies tend to capture the increasing relation between `SPoCP` and V_0 implied by straight bonds.[23] `SPoCP` curves are observed in our quantitative experiments illustrated in **Fig. 9** to change from humped shape to positive sloping with the decrement of the early redemption likelihood that can be influenced by four factors: the levels of interest rates, the lengths of call protection periods and time to maturities, and the levels of call prices. By combining our experiments with the descriptions of LT's and KM's data sets, we find some explanations for their conflicts.

Increasing the interest rate level r could reduce the likelihood of the early redemption and hence change the `SPoCP`-V_0 relation from the hump-shaped pattern (denoted by solid curves) to the posi-

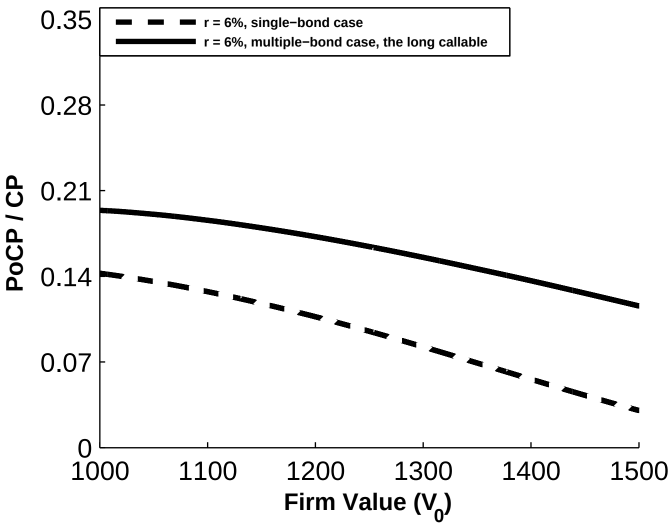
[21] It is not illustrated in **Fig. 8(a)** so we can zoom in and compare another three call boundaries.

[22] Note that the bond would eventually depreciate due to the default risk if the firm's creditworthiness deteriorated further.

[23] Note that the value of a straight bond increases monotonically with the firm's creditworthiness.



(a)



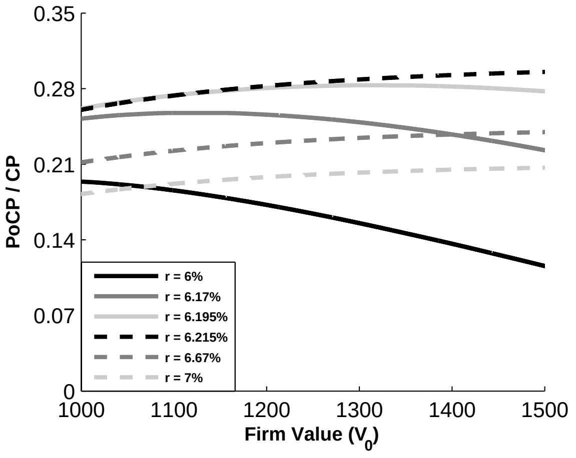
(b)

Figure 8: Measuring Call Delay Phenomenon with Call Boundaries and SPoCP. The five-bond debt structure is identical to the structure studied in Fig. 4 except that both B_2 and B_3 are set as callable bonds with a call price \$100. The single-bond debt structure contains only one 16-year callable bond B' with a face value 500 and a coupon rate 10% that equal to the total amount of outstanding bonds and the coupon rate of B_3, respectively, in the five-bond debt structure. Panel (a) measures the call boundary in terms of the prevailing firm value V_5 and the interest rate level r at year 5. For example, it is optimal to redeem (hold) B' in the one-bond structure if the firm value V_5 exceeds (is lower than) the dashed curve. It is optimal to redeem (hold) B_3 and B_2 in the five-bond structure if V_5 exceeds (is lower than) the black solid and the gray solid curves, respectively. Panel (b) displays the call boundaries for B' and B_3 by the dashed and the solid curves, respectively, given the prevailing interest rate $r = 6\%$. It is optimal to redeem (hold) B' and B_3 when SPoCP exceeds (is lower than) their boundaries. Other numerical settings are $\sigma = 20\%$, $\tau = 35\%$ and $\omega = 50\%$.

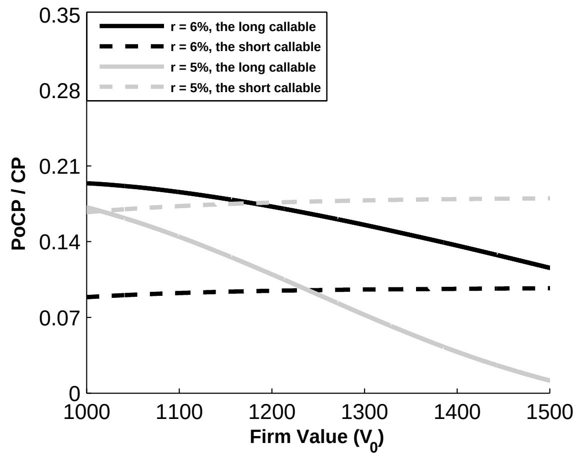
tively related pattern (denoted by dashed curves) as illustrated in **Fig. 9(a)**. Although it is hard to accurately measure the average interest rate levels of LT's and KM's data sets, we can measure the average 10-year Treasury yields of their sample periods as shown in **Fig. 10** instead. The callable bonds sampled by the former study are from August 1991 to August 1992, whereas the bonds sampled by the latter study are from January 1975 to March 1994. The average 10-year Treasury yields for the former sample period 7.31% is significantly lower than that for the latter sample period 9.22%. Since a higher interest level implies that callable bonds are less likely to be redeemed early and that SPoCP-V_0 tends to form a positive relationship as illustrated in **Fig. 9(a)**, this might explains why LT and KM produce different SPoCP-V_0 relations.

There are two other possible reasons that cause KM to render humped-shape relations insignificant. First, they claim that the average call protection period of their callable bonds samples accounts for more than 65% (6.41 years out of 9.83 years) of the average maturity of these bonds.[24] Our experiment shows that increasing the call protection period could decrease the early redemption likelihood and change the SPoCP-V_0 relation from the hump-shaped pattern to the positively related one as illustrated in **Fig. 9(b)**. Second, the long-term callable bond is prone to be redeemed early than the short-term

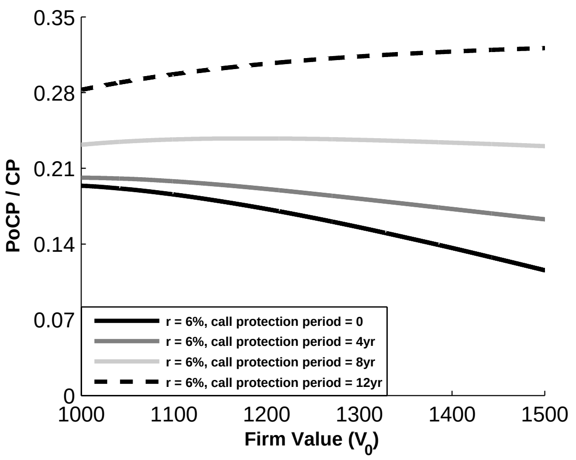
[24]The information about the call protection period refers to King (2002).



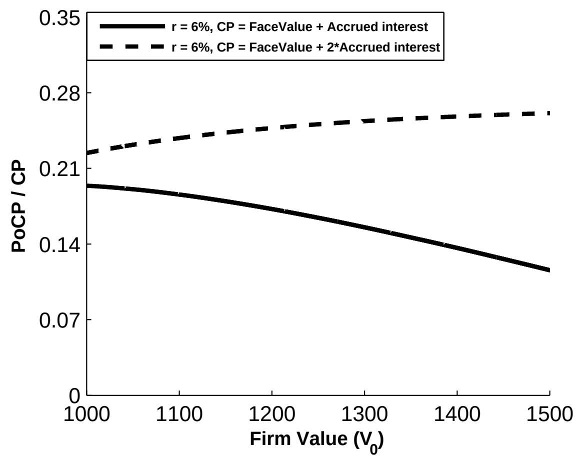
(a) Interest Rate Levels



(c) Lengths of Time to Maturities



(b) Lengths of Call Protection Periods



(d) Levels of Effective Call Prices

Figure 9: Sensitivity Analyses of the Relation between SPoCP and V_0. The debt structure is identical to the five-bond structure studied in Fig. 8. The relation of the 16-year callable bond's SPoCP and V_0 are drawn in panels (a), (b) and (d). Panel (c) plots SPoCPs for coexisting 10-year (short-term) and 16-year (long-term) callable bonds. The solid and the dashed patterns denote that the curves are hump-shaped and upward-sloping, respectively. All other numerical settings are the same as those in Fig. 8 (b) unless stated otherwise.

one in a complicated debt structure with multiple outstanding bonds as illustrated in Fig. 8 (a). The differences of early redemption likelihoods lead to hump-shaped and positive SPoCP-V_0 relation for long-term and short-term callable bonds, respectively, as illustrated in **Fig. 9(c)**. Note that KM seems to sample many callable bonds by issuers with multiple outstanding callable bonds.[25] That may be another reason for the insignificant hump-shaped relation concluded in their study.

In additional to the aforementioned factors that could change the SPoCP-V_0 relation, our experiment in panel (d) also shows that the increment of the CP would also decrease the early redemption likelihood and tend to produce positive relation. Generally speaking, our quantitative analyses can

[25] According to the report in KM, among the 1642 calls made by 530 firms, "294 firms called (possibly multiple bonds) one time, 108 firms called bonds two different times, 30 firms called three different times, 33 firms called four times and 21 firms called more than four times during the sample period."

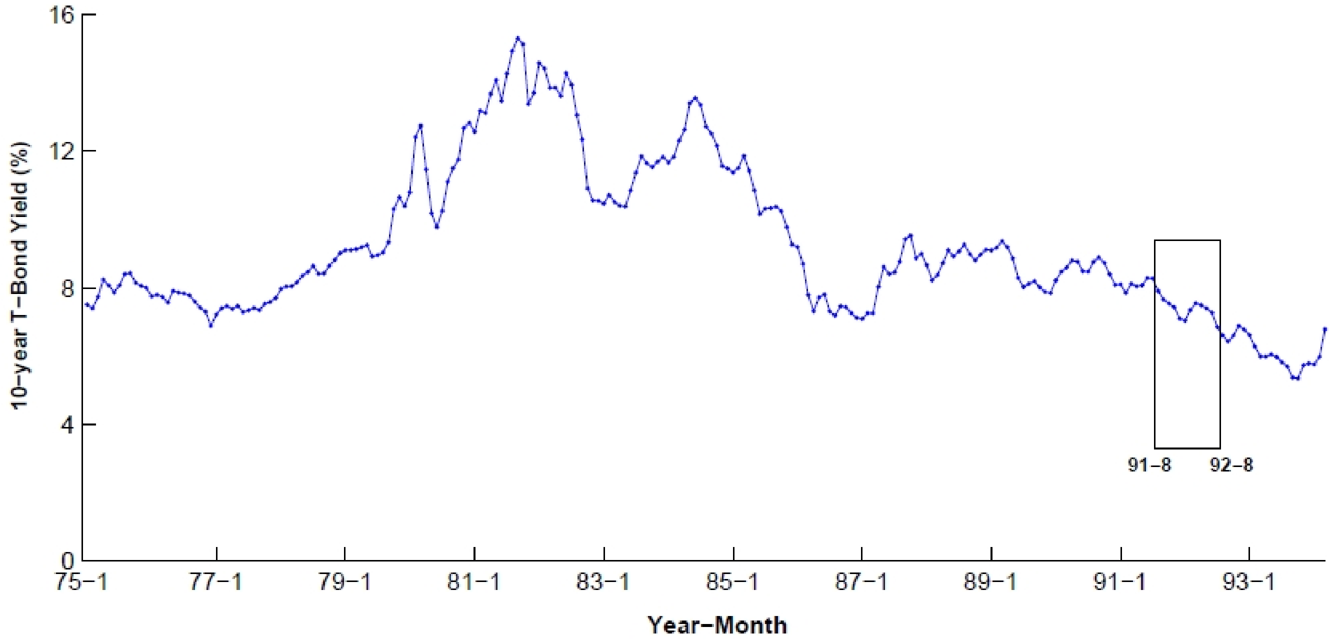


Figure 10: 10 Year Treasury Yields as Proxies for Interest Rate Levels. The whole time span and the time span marked by the rectangle denote the sample periods of LT's and KM's data sets, respectively.

not only provide theoretical insights for the phenomena or conflicts found in empirical studies, but quantitatively measure (or predict) the impacts of various factors on the dependent variables.

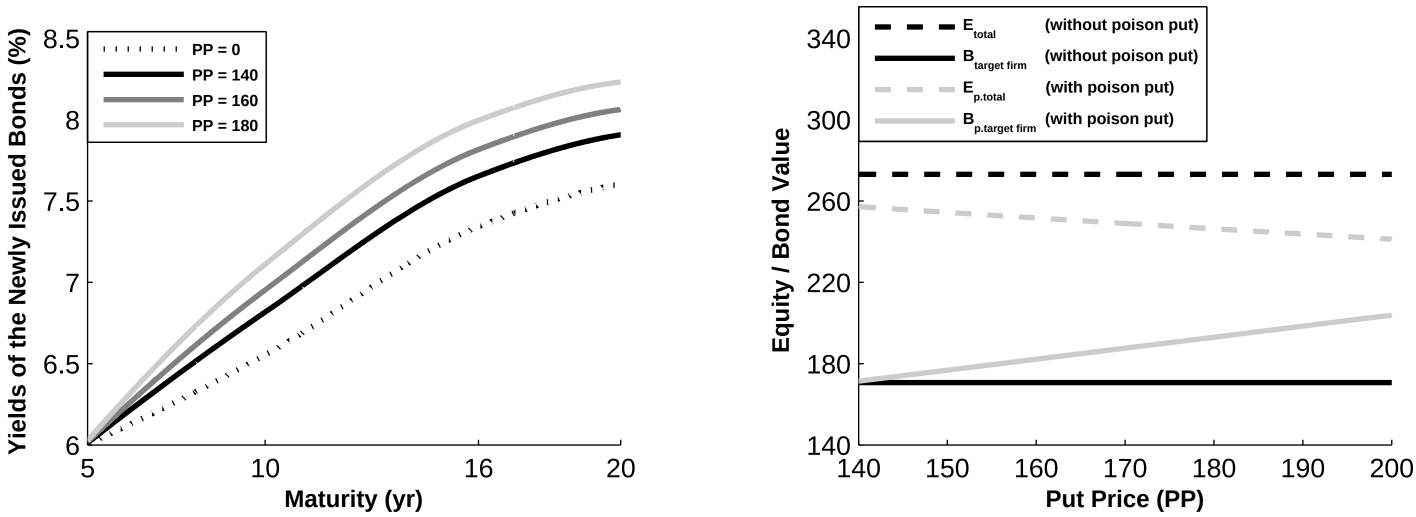
4.4 Poison Puts and the Bidder's Costs of Debt Financing

A poison put is a bond covenant that gives bondholders the right to demand redemption at a pre-specified put price `PP` prior to maturity in case a certain event, like a leveraged buyout (`LBO`), happens. It can increase the bidder's cost for raising new debts, have negative effect on equity holders, and protect bondholders of the target firm as argued in Cook and Easterwood (1994). This is because executing the poison put can alter the bidder's loan repayment schedule to improve the effective payment priority of the target firm's bond at the expense of other outstanding debts of the bidder. Cremers et al. (2007) claim that bond holders' governance through poison put covenants can efficiently mitigate the asset substitution problem.

Our forest model can quantitative analyze how a poison put can avoid the asset substitution action like `LBO` as illustrated by a hypothetical scenario in **Fig 11**. Consider a poison put that can require the bidder to redeem a 30-year bond originally issued by the target firm once the bidder's asset value after the `LBO` falls below the sum of face values of its outstanding bonds.[26] The bidder issues serial bonds to finance `LBO`, and the impact of different levels of `PP` on the yield spreads are illustrated in panel (a). We also illustrate the case that the 30-year bond is not protected by the poison put (denoted by `PP=0`) for comparisons. Potential premature redemptions significantly raises the yields of the newly issued serial bonds (by comparing the dot curve with solid curves), because it changes the bidder's debt repayment order and deteriorate the effective priorities of serial bonds. The magnitude of `PP` can be viewed as the level of protection for the 30-year bond at the expense of holders of serial bonds and equity. It can be observed that the yield curve of serial bonds increase with `PP`. **Fig 11(b)**

[26]This positive net worth covenant is popular adopted in academic literature like Leland (1994).

illustrates the impacts of PP's magnitude on the values of equity and the 30-year bond denoted by gray dashed and gray solid curves, respectively. The values of the equity and the bond without the poison put protection are denoted by the black dashed and the black solid horizontal lines for comparisons. It can be observed that the increment of PP would increase the protection for the 30-year bond (by comparing the black solid lines with gray solid lines) and deprive the wealth of equity holders (by comparing the black dashed lines with gray dashed lines) due to high cost for raising debt capital reflected by the yields of serial bonds.



(a) (b)

Figure 11: The Impacts of the Poison Put. The target firm is assumed to have one 30-year outstanding bond with the face value 100 and the coupon rate 14%. The bidder finances LBO by issuing another 4 otherwise identical serial bonds with face value 100 and maturity 5, 10, 16, and 20 years. After the LBO, all serial bonds are senior to the 30-year bond. Panel (a) displays the change of yield spread curves for serial bonds with the change of PP. In panel (b), $\mathtt{E}_{\mathtt{p.total}}$ and $\mathtt{E}_{\mathtt{total}}$ denote the total equity value after the LBO with or without the poison put embedded in the 30-year bond, respectively. $\mathtt{B}_{\mathtt{p.target\ firm}}$ and $\mathtt{B}_{\mathtt{target\ firm}}$ denote the value of the 30-year bond with or without the poison put, respectively. Other numerical settings are $V_0 = 1000$, $r = 6\%$, $\sigma = 20\%$, $k = 1\%$, $\tau = 35\%$ and $\omega = 50\%$.

5 Conclusion

If the market participants recognize debt heterogeneity, then it is nature to anticipate that market prices would account for this fact. Selectively simplifying this recognition probably contributes to biased estimations. This article develops a structural model based on the compound option approach to study the impacts of an issuer's complicated debt structure on not only the values of corporate securities but the relevant claim holders' decisions simultaneously from four observable facets of its debt structure: the leverage ratio, maturity structure, priority structure and covenant structure. To achieve this analysis framework, we incorporate the debt-structure-dependent default trigger and propose a novel quantitative method — the forest, to capture the contingent changes in debt structure due to premature redemptions driven either compulsorily by event-trigger covenants or voluntarily by callability provisions. This numerical method can not only solve the unsolved problem in Jones et al. (1983) but allow us to investigate the bond issuer's optimal call policy when the debt structure is complicated.

Compared with the results produced by extant structural models, ours are more consistent with the observations documented in empirical studies. Besides the spread-rate, spread-firm value volatility and the spread-leverage relation, our framework robustly generates upward-sloping yield spread curves for either investment- or speculative-grade issuers, whereas numerous existing models may predict hump-shaped or downward-sloping curves. Second, our framework predicts higher yield spreads for not only the existing long-term bonds but also the whole term structure of the existing bonds in the firm with a larger proportion of bonds that are about to be rolled over, and such effect will be amplified during the recession times, when the issuing firm has poor creditworthiness or once the maturing bond is replaced by another short-term bond. Third, our framework predicts that the improvement in bond seniority decreases the bond yield spreads, and the dilution effect of a short-term junior bond issuance on an existing long-term senior bond may be mitigated by including the payment blockage covenant in the senior bond. Fourth, our framework forecasts longer call delay because we jointly consider the impacts of tax shield benefits, interaction effect and the wealth transfer effect on the firm's optimal call policy, and successfully reconciles the conflicts between the theoretical predictions and empirical observations that proxy the wealth transfer effect through four aspects: (i) different effective call prices; (ii) different levels of call protection; (iii) different interest rate levels; (iv) the simultaneous presence of callable bonds with different remaining time to maturity. Finally, our framework displays salient effect of including poison put covenants on the value of target bond holders and the bidder's costs of debt financing for a LBO. It thus appears that taking an issuer's debt structure into account has a significant impact on corporate bond valuation, and it helps to reconcile many of the predictions by extant structural models with empirical observations.

References

Acharya, v. and J. Carpenter (2002). Corporate bond valuation and hedging with stochastic interest rates and endogenous bankruptcy. *Review of Financial Studies 15*(5), 1355–1383.

Attaoui, S. and P. Poncet (2013). Capital structure and debt priority. *Financial Management 42*(4), 737–775.

Avramov, D., G. Jostova, and A. Philipov (2007). Understanding changes in corporate credit spreads. *Financial Analysts Journal*, 90–105.

Barclay, M. J., L. M. Marx, and C. W. Smith Jr (2003). The joint determination of leverage and maturity. *Journal of Corporate Finance 9*(2), 149–167.

Black, F. and J. Cox (1976). Valuing corporate securities: Some effects of bond indenture provisions. *Journal of Finance 31*(2), 351–367.

Brennan, M. J. and E. S. Schwartz (1977). Convertible bonds: Valuation and optimal strategies for call and conversion. *Journal of Finance 32*(5), 1699–1715.

Bris, A., I. Welch, and N. Zhu (2006). The costs of bankruptcy: Chapter 7 liquidation versus chapter 11 reorganization. *Journal of Finance 61*, 1253–1303.

Briys, E. and F. De Varenne (1997). Valuing risky fixed rate debt: An extension. *Journal of Financial and Quantitative Analysis 32*(2), 239–248.

Broadie, M., M. Chernov, and S. Sundaresan (2007). Optimal debt and equity values in the presence of chapter 7 and chapter 11. *Journal of Finance 62*(3), 1341–1377.

Broadie, M. and O. Kaya (2007). A binomial lattice method for pricing corporate debt and modeling chapter 11 proceedings. *Journal of Financial and Quantitative Analysis 42*, 1253–1303.

Chen, H. (2010). Macroeconomic conditions and the puzzles of credit spreads and capital structure. *Journal of Finance 65*(6), 2171–2212.

Chen, H., Y. Xu, and J. Yang (2013). Systematic risk, debt maturity, and the term structure of credit spreads. *Working Paper*.

Chen, N. and S. Kou (2009). Credit spreads, optimal capital structure, and implied volatility with endogenous default and jump risk. *Mathematical Finance 19*(3), 343–378.

Colla, P., F. Ippolito, and K. Li (2013). Debt specialization. *Journal of Finance 68*(5), 2117–2141.

Collin-Dufresne, P. and R. S. Goldstein (2001a). Do credit spreads reflect stationary leverage ratios? *Journal of Finance 56*(5), 1929–1957.

Collin-Dufresne, P., R. S. Goldstein, and J. S. Martin (2001b). The determinants of credit spread changes. *Journal of Finance 56*(6), 2177–2207.

Cook, D. O. and J. C. Easterwood (1994). Poison put bonds: An analysis of their economic role. *Journal of Finance 49*(5), 1905–1920.

Cox, J. C., S. A. Ross, and M. Rubinstein (1979). Option pricing: A simplified approach. *Journal of Financial Economics 7*(3), 229–263.

Cremers, K. M., V. B. Nair, and C. Wei (2007). Governance mechanisms and bond prices. *Review of Financial Studies 20*(5), 1359–1388.

Crosbie, P. and J. Bohn (2002). Modeling default risk. *Working Paper KMV Corp.*.

Dai, T.-S. and Y.-D. Lyuu (2010). The bino-trinomial tree: A simple model for efficient and accurate option pricing. *Journal of Derivatives 17*(4), 7–24.

Davydenko, S. A. (2007). When do firms default? a study of the default boundary. *Working Paper (August 2007)*.

Davydenko, S. A. (2012). When do firms default? a study of the default boundary. *Working Paper (November 2012)*.

Duffee, G. R. (1998). The relation between treasury yields and corporate bond yield spreads. *Journal of Finance 53*(6), 2225–2241.

Duffie, D. (1996). *Dynamic asset pricing theory.* Princeton University Press.

Eom, Y. H., J. Helwege, and J.-z. Huang (2004). Structural models of corporate bond pricing: An empirical analysis. *Review of Financial Studies 17*(2), 499–544.

Fama, E. F. and M. H. Miller (1972). *The Theory of Finance*, Volume 3. Dryden Press Hinsdale, IL.

Fan, H. and S. M. Sundaresan (2000). Debt valuation, renegotiation, and optimal dividend policy. *Review of Financial Studies 13*(4), 1057–1099.

Figlewski, S. and B. Gao (1999). The adaptive mesh model: a new approach to efficient option pricing. *Journal of Financial Economics 53*(3), 313–351.

Flannery, M. J., S. S. Nikolova, and Ö. Öztekin (2012). Leverage expectations and bond credit spreads. *Journal of Financial and Quantitative Analysis 47*(04), 689–714.

François, P. and E. Morellec (2004). Capital structure and asset prices: Some effects of bankruptcy procedures*. *Journal of Business 77*(2), 387–411.

Galai, D., A. Raviv, and Z. Wiener (2007). Liquidation triggers and the valuation of equity and debt. *Journal of Banking & Finance 31*(12), 3604–3620.

Geske, R. (1977). The valuation of corporate liabilities as compound options. *Journal of Financial and Quantitative Analysis 12*(04), 541–552.

Gopalan, R., F. Song, and V. Yerramilli (2014). Debt maturity structure and credit quality. *Journal of Financial and Quantitative Analysis, forthcoming*.

Gorton, G. and J. Kahn (2000). The design of bank loan contracts. *Review of Financial Studies 13*(2), 331–364.

Hackbarth, D. and D. C. Mauer (2012). Optimal priority structure, capital structure, and investment. *Review of Financial Studies 25*(3), 747–796.

He, Z. and W. Xiong (2012). Rollover risk and credit risk. *Journal of Finance 67*(2), 391–430.

Helwege, J. and C. M. Turner (1999). The slope of the credit yield curve for speculative-grade issuers. *Journal of Finance 54*(5), 1869–1884.

Huang, J.-z. and M. Huang (2012). How much of the corporate-treasury yield spread is due to credit risk? *Review of Asset Pricing Studies 2*(2), 153–202.

Huang, J.-Z. and X. Zhang (2008). The slope of credit spread curves. *Journal of Fixed Income 18*(1), 56–71.

Ingersoll, J. E. (1977a). A contingent-claims valuation of convertible securities. *Journal of Financial Economics 4*(3), 289–321.

Ingersoll, J. E. (1977b). An examination of corporate call policies on convertible securities. *Journal of Finance 32*(2), 463–478.

Ingersoll, J. E. (1987). *Theory of financial decision making*, Volume 3. Rowman & Littlefield.

Jacoby, G. and I. Shiller (2010). Corporate bond pricing and the effects of endogenous default and call options. *Journal of Fixed Income 20*(2), 80–100.

Jensen, M. C. and W. H. Meckling (1976). Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics 3*(4), 305–360.

Jones, E. P., S. P. Mason, and E. Rosenfeld (1983). Contingent claims valuation of corporate liabilities: theory and empirical tests. *National Bureau of Economic Research Working Paper*.

Jones, E. P., S. P. Mason, and E. Rosenfeld (1984). Contingent claims analysis of corporate capital structures: An empirical investigation. *Journal of Finance 39*(3), 611–625.

Ju, N. and H. Ou-Yang (2006). Capital structure, debt maturity, and stochastic interest rates*. *Journal of Business 79*(5), 2469–2502.

Kahl, M., A. Shivdasani, and Y. Wang (2015). Short-term debt as bridge financing: Evidence from the commercial paper market. *Journal of Finance 70*(1), 211–255.

Kim, I., K. Ramaswamy, and S. Sundaresan (1993). Does default risk in coupons affect the valuation of corporate bonds?: A contingent claims model. *Financial Management 22*(3), 117–131.

King, T.-H. D. (2002). An empirical examination of call option values implicit in us corporate bonds. *Journal of Financial and Quantitative Analysis 37*(04), 693–721.

King, T.-H. D. and D. C. Mauer (2000). Corporate call policy for nonconvertible bonds*. *Journal of Business 73*(3), 403–444.

Kuehn, L.-A. and L. Schmid (2014). Investment-based corporate bond pricing. *Journal of Finance, forthcoming*.

Lando, D. (2004). *Credit risk modeling: theory and applications.* Princeton University Press.

Leland, H. E. (1994). Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance 49*(4), 1213–1252.

Leland, H. E. (2004). Predictions of default probabilities in structural models of debt. *Working paper*.

Leland, H. E. and K. B. Toft (1996). Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *Journal of Finance 51*(3), 987–1019.

Linn, S. C. and D. R. Stock (2005). The impact of junior debt issuance on senior unsecured debt's risk premiums. *Journal of Banking and Finance 29*(6), 1585–1609.

Liu, S. X., H. Qi, and C. Wu (2006). Personal taxes, endogenous default, and corporate bond yield spreads. *Management Science 52*(6), 939–954.

Longstaff, F. A. and E. S. Schwartz (1995). A simple approach to valuing risky fixed and floating rate debt. *Journal of Finance 50*(3), 789–819.

Longstaff, F. A. and B. A. Tuckman (1994). Calling nonconvertible debt and the problem of related wealth transfer effects. *Financial Management 23*(4), 21–27.

Mauer, D. C. (1993). Optimal bond call policies under transactions costs. *Journal of Financial Research 16*(1), 23–37.

Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates*. *Journal of Finance 29*(2), 449–470.

Modigliani, F. and M. H. Miller (1958). The cost of capital, corporation finance and the theory of investment. *American Economic Review 48*(3), 261–297.

Myers, S. C. (1977). Determinants of corporate borrowing. *Journal of Financial Economics 5*(2), 147–175.

Nagler, F. (2014). Rolling over corporate bonds: How market liquidity affects credit risk. *Working Paper*.

Ou, S., D. Chlu, and A. Metz (2011). Corporate default and recovery rates, 1920–2010. *Moody's Investors Service*.

Rauh, J. D. and A. Sufi (2010). Capital structure and debt structure. *Review of Financial Studies 23*(12), 4242–4280.

Smith, C. W. and J. B. Warner (1979). On financial contracting: An analysis of bond covenants. *Journal of Financial Economics 7*(2), 117–161.

Thatcher, J. S. (1985). The choice of call provision terms: evidence of the existence of agency costs of debt. *The Journal of Finance 40*(2), 549–561.

Wang, C.-J., T.-S. Dai, and Y.-D. Lyuu (2014). Evaluating corporate bonds with complicated liability structures and bond provisions. *European Journal of Operational Research 237*(2), 749–757.

Welch, I. (1997). Why is bank debt senior? a theory of asymmetry and claim priority based on influence costs. *Review of Financial Studies 10*(4), 1203–1236.

Xu, Q. (2014). Kicking the maturity down the road: Early refinancing and maturity management in the corporate bond market. *Working Paper*.

Zhou, C. (2001). The term structure of credit spreads with jump risk. *Journal of Banking and Finance 25*(11), 2015–2040.

Appendix A Forests in More Complex Forms

To explore how a forest framework can grow in a more complex form to deal with complicated debt structures and covenants, the following scenarios will be studied based on a generic three-bond example. Bonds B_1, B_2, and B_3 mature at time T_1, time T_2, and time T_3, respectively. Note that coupon payments are ignored in the following forest structure constructions for simplicity.

A.1 Refunding a Callable Bond

The method to construct a forest for dealing with non-refundable callable bonds studied in Section 3.3 can be slightly modified for dealing with refundable callable bonds as illustrate in Fig. 12. Consider the case that B_2 is a refundable callable bond which can be called back using the proceeds from issuing a new bond B_2' with the same maturity date T_2. On each call date, the issuer can decide whether it should refund B_2 or not under the burdens for repaying the straight bonds B_1 and B_3. Thus the forest is composed of two trees; the upper (the lower) layer tree models the scenario that B_2 is not yet refunded (already refunded by issuing B_2'). These two trees can be constructed by following the methods described in Section 3.1 described as follows.

In the upper tree, the issuer should repay the coupons plus the principal payments of B_1, B_2, and B_3 at these bonds's maturities. The concept of the remaining asset is implemented by subtracting the loan repayments $C^O_{T_1}$ and $C^O_{T_2}$ (denoted by downward arrows) from $V_{T_1^-}$ and $V_{T_2^-}$ (denoted by dashed circles) to get the remaining value V_{T_1} and V_{T_2} (denoted by boldfaced circles), respectively. The firm defaults if its asset value can not meet the debt-structure-dependent default boundaries Θ_{T_1}, Θ_{T_2}, and Θ_{T_3} (plotted by gray thick lines), which are equal to the values of frozen asset plus the repayment at these three repayment dates. Tree nodes I, J, and K are decided to match the boundaries to avoid unstable pricing results due to nonlinearity errors studied in Figlewski and Gao (1999). The lower layer tree follows the similar structure except that the payment to the callable bond B_2 is replaced by the payment to the new bond B_2' (denoted by $C^O_{T_2}$). The default boundary Θ_{T_2} is redefined as the values of frozen asset plus the repayments to the bond B_2'. Note that tree nodes E, G, and H are decided to match default boundaries to avoid unstable pricing results.

Calling back B_2 using the proceeds from issuing B_2' at a node, says U, would change the prevailing debt structure and can be modeled by transferring from the upper layer tree, to the lower layer one. Refunding would change the firm asset value (i.e., $v(U)-v(W)$) by $\mathtt{CP}-\mathtt{BV}_2$ (marked by the downward arrow from node U to W), where $\mathtt{CP}$ denotes the effective call price and $\mathtt{BV}_2$ denotes the issuance price of B_2'. The trinomial structure emitted from the node W to nodes X, Y, and Z is constructed by following Dai and Lyuu (2010) method to simulate the firm value dynamics during the time interval $[t, t+\Delta t]$. Note that the issuance price $\mathtt{BV}_2$ should be equal to the value B_2' calculated by the backward induction at node W. A proper solution for $\mathtt{BV}_2$ can be numerically solved by root finding instructions like "fsolve" in Matlab. Besides, the aforementioned process can be slightly modified to consider B_2' issuance cost as stated in Mauer (1993). Let each issuance incurs a fixed cost ν_F plus μ_V proportion of the issuance price. Then the change of the firm asst value (i.e., $v(U)-v(W)$) due to refunding is expressed as $(\mathtt{CP}-BV_2)+(\mu_F+\mu_V BV_2)$ instead.

To maximize the equity value, the issuer would find the optimal strategy for refunding B_2 with the new issuance B_2' at each B_2's call date as the method for making the optimal call decision discussed in Section 3.3.2. Now we use node U in the upper layer tree to demonstrate this process. If the issuer refunds B_2 at node U, the "**r**efunded" equity value $E^R(t^-, v(U))$ can be evaluated as the dividend paid at node U plus the continuation value evaluated by applying the backward induction on the successor nodes X, Y, and Z in the lower layer tree to reflect the replacement of B_2 with B_2'. Otherwise, B_2 is alive and the "**n**on-refunded" equity value $E^N(t^-, \upsilon(U))$ can be evaluated as the dividend paid at node U plus the continuation value evaluated by applying the backward induction on the successor nodes R and S in the upper layer tree. If $E^R(t^-, \upsilon(U))$ is larger than $E^N(t^-, \upsilon(U))$, the issuer refunds B_2 by issuing B_2' and the continuation values of equity, B_1, B_2', and B_3 are calculated by applying the backward induction on nodes X, Y, and Z. Otherwise, the issuer would not call B_2 back and the continuation values of all claims (except B_2') are calculated by applying the backward induction on nodes R and S.

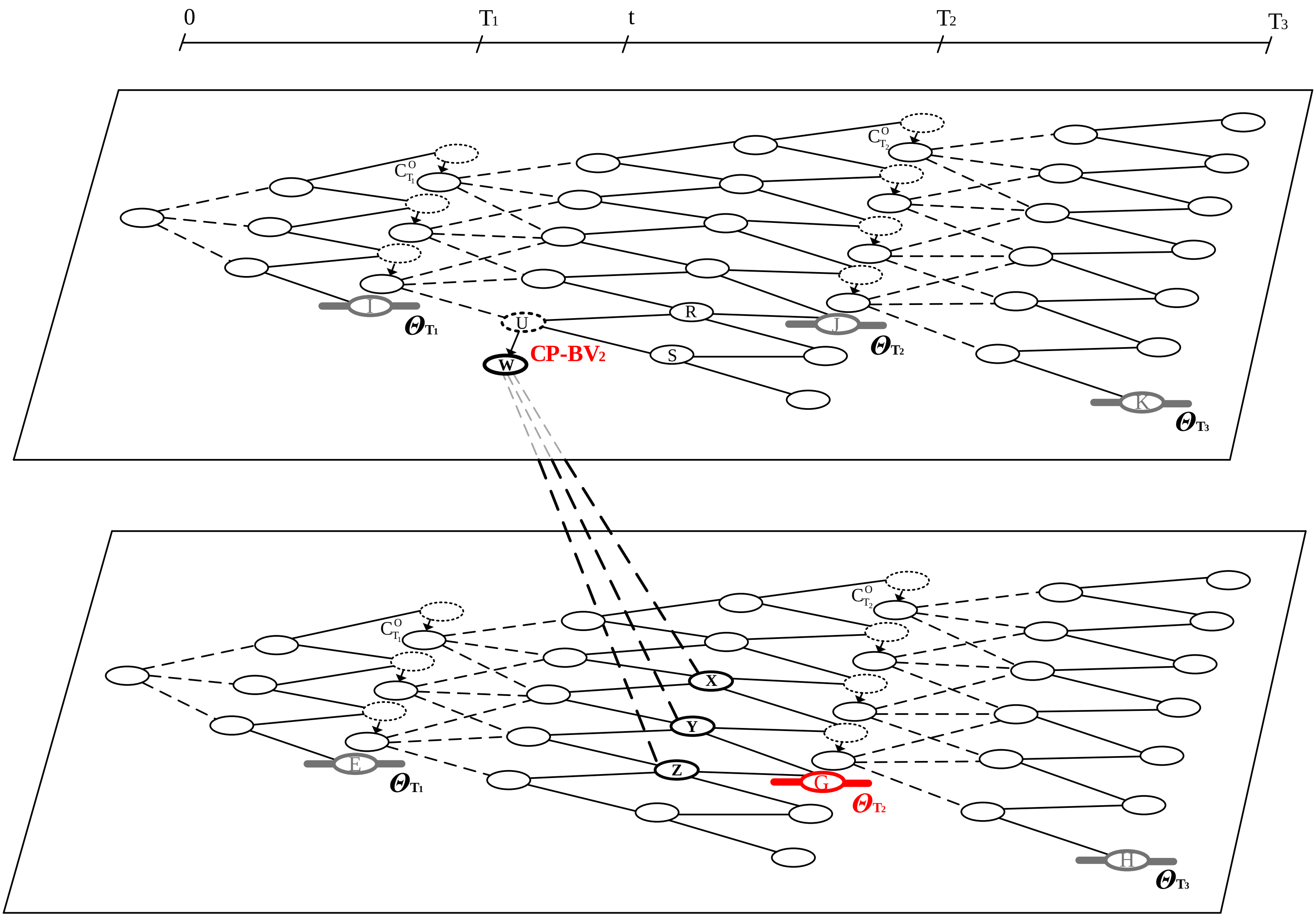


Figure 12: A Two-Layer Forest for Modeling a Callable Bond Refunding. In this generic three-bond scenario, B_1, B_2, and B_3 will mature at time T_1, T_2, and T_3, respectively. The forest is composed of two trees; the upper (lower) layer tree models the scenario that B_2 is not yet refunded (already refunded by issuing B_2'). Refunding B_2 would change the issuing firm's asset value (denoted by a downward jump from node U to W) and the prevailing debt structure (denoted by transferring from the upper layer tree to the lower one.) The terms colored in red (including $\mathtt{BV}_2$,$C^O_{T_2}$, andΘ_{T_2}) denote that they depend on the payment/value of the new issuance B_2'. Cox et al. (1979)'s binomial structures are plotted by the solid lines and Dai and Lyuu (2010)'s trinomial structures are plotted by dashed lines. The boldfaced circles denote the remaining assets after loan repayments. Node I, J, K, E, G and H are decided to exactly match the default boundaries marked by thick lines.

A.2 Multiple Callable Bonds

To analyze early redemptions of one or some callable bonds from a complicated debt structure with multiple callable bonds, we can model possible debt structure transitions due to different redemption strategies by a forest with multiple layers. Now we consider a slight modification of the aforementioned three-bond example by setting B_1 and B_2 as non-refundable callable bonds. The resulting forest illustrated in **Fig. 13** can be decomposed into three layers with four trees. The upper layer contains only one tree (marked as **State 1**) which models the dynamics of the firm's asset value under the condition that both B_1 and B_2 are not called yet. The middle layer contains two trees marked by **State 2** and **3**. The former (latter) tree models the dynamics of the asset value given that only B_2 (B_1) is called. The lower layer contains one tree (marked as **State 4**) which models the scenario that both B_1 and B_2 are already called. The structure of each tree is constructed by following the method in Section 3.1 to calibrate the payment schedule and the default boundary implied by the tree's corresponding debt structure. For example, the loan repayment at time T_2 (T_1) is missing in the tree **State 2** (**State 3**) since B_2 (B_1) is called in that tree. The loan repayments at both time T_1 and T_2 are missing in the tree **State 4** since both B_1 and B_2 are all called.

Each transition between two trees from different layers denotes the change of the debt structure due to an early redemption of callable bond(s). The transition is modeled by a change of the firm asset value (marked by a downward jump from a dotted circle to a boldfaced one) to reflect the redemption payment and by a trinomial structure connecting to successor nodes in another tree to reflect the change of the debt structure. The transition structure can be constructed for each tree node[27] at a call date by following the method in Section 3.3.1.

The issuing firm will select the optimal bond redemption strategies to maximize the value of equity holders as described in Section 3.3.2. The values of bonds and equity under different debt structures can be evaluated by applying backward induction on their corresponding trees. Therefore, at each node located at a call date, the issuer can pick the best redemption strategy by comparing their corresponding equity values. Take the redemption decision for a node at the tree **State 1** for example, the issuer may choose not to redeem any callable bonds, to redeem B_1 and B_2 simultaneously (i.e., transiting from **State 1** to **State 4**), or to redeem either B_1 or B_2 only (i.e., from **State 1** to either **State 3** or **State 2**). Note that the forest structure can also analyze the benefits of sequential redemptions. For example, the issuer may first redeem B_1 and then redeem B_2 (i.e., transiting from **State 1** to **State 3** and then to **State 4**), or the opposite (i.e., transiting from **State 1** to **State 2** and then to **State 4**). The analyses of sequential redemptions can explain call strategies and in consequence the call delay phenomena (see Section 4.3) found in past empirical studies like King and Mauer (2000).

Appendix B Robustness Checks

Checking whether a quantitative framework produces accurate and stable pricing results is critical before applying the framework to compare or to analyze empirical observations. Although pricing results generated by tree methods should converge to theoretical values with the increment of the number of the time steps of the tree or forest (see Duffie, 1996), but inappropriate tree structure adjustments or backward inductions can destroy the accuracy. Previous literature like Broadie and Kaya (2007) and Wang et al. (2014) examines the robustness of their methods by showing that their

[27] It does not include the nodes in **State 4** since no callable bonds exist in its corresponding debt structure.

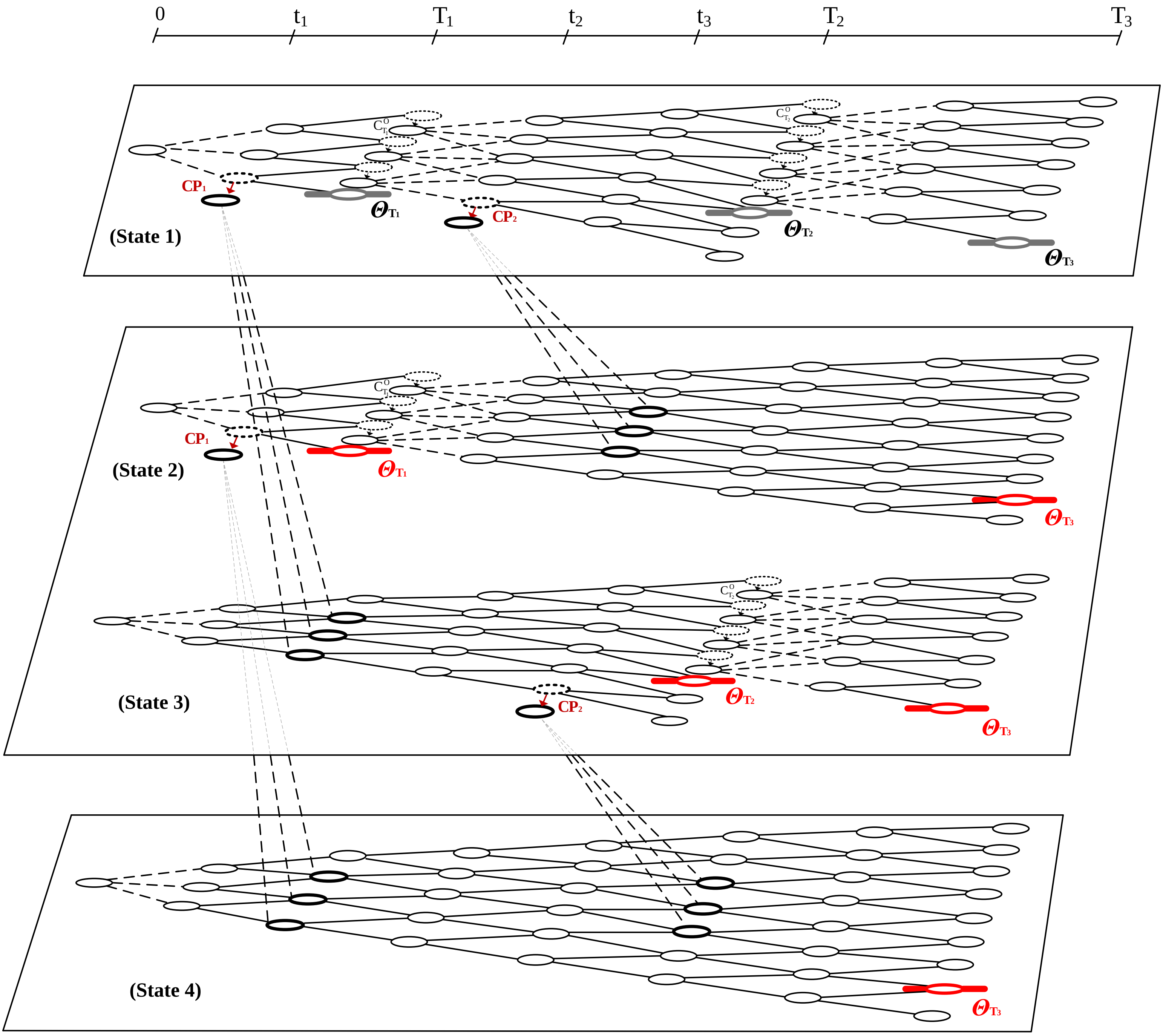


Figure 13: A Three-Layer Forest for Modeling Possible Redemption Scenarios for Two Callable Bonds. The debt structure settings are the same as those in Fig. 12 except that B_1 and B_2 are non-refundable callable bonds. The forest is composed of three layers with four trees marked by **State 1** $\cdots$ **State 4**. The CRR binomial branches are plotted by solid lines and the Dai and Lyuu (2010)'s trinomial branches are plotted by dashed lines. The gray/red nodes are decided to exactly match the default boundaries marked by thick gray/red lines.

pricing results converge to those generated by analytical formulas under certain simple debt structures. However, analytical formulas are not available for complicated debt structure scenarios and hence the correctness for their complex tree implementations is hard to be verified.

Instead of directly verifying the accuracy of each pricing result, we suggest to indirectly check the rationality of the pricing results in whole by taking advantage of the capital structure irrelevance theory proposed by Modigliani and Miller (1958). Essentially, in a perfect and frictionless market, the market value of a firm is not influenced by the capital structure used to finance its operations and our framework should produce the same market value regardless the changes of debt structures under otherwise identical conditions. Indeed, given that the capital market is frictionless (i.e, no bond issuance costs, taxes, and bankruptcy costs), our experiment in **Table 2** shows that the levered firm value, which is equal to the lump sum of all contingent claims' values generated by our framework, is essentially equal to the initial firm asset value V_0 (1000 in this experiment) under different debt structure scenarios listed in the first row. The "Straight Bonds" scenario assume that all outstanding bonds are equal-priority straight bonds. It can be observed that the lump sum of these five bond values listed in the second column (i.e., 103.85, 110.62, ..., 180.16) plus the equity value 339.17 is about 1000. By checking our numerical results in other columns, we can verify that this irrelevant property also holds under other settings.

In the "Payment Blockage" scenario, the payments to the junior bond B_2 occurred within one year prior to default (i.e., the length of the blockage period η is one year) can be blocked to fulfill the payments of the other unmatured senior bonds. This scenario can be evaluated by the method discussed in **Section 3.2**. By comparing this scenario with the otherwise identical scenario "Straight Bonds", the payment blockage covenants apparently increase the values (decrease the yield spreads listed in parentheses) of bonds matured later than the junior bond B_2 (i.e., B_3, B_4 and B_5) at the expense of B_2's value.

In the "Multiple Callable Bonds" scenario, all settings are the same as those in the "Straight Bond" scenario except that B_2 and B_3 are callable bonds. The forest method discussed in Section 3.3 and Appendix A.2 can be applied to analyze complex call strategies that are claimed to be intractable in Jones et al. (1983). Compared with the "Straight Bonds" scenario, the call options embedded in B_2 and B_3 decrease their bond values (increase the bond yield spreads) and benefit the holders of equity and bonds matured after B_3.

Besides, tree (or forest) methods may produce oscillating pricing results as in Fig. 5 and 6 in Broadie and Kaya (2007) due to the nonlinearity error problem (see Figlewski and Gao (1999)). Oscillating numerical errors may interfere the analyses of the bond covenants' impacts and decisions of claims' holders. For example, the impact of the payment blockage covenant on B_1's value is insignificant and cannot be clearly identified in our experiment. Specifically, B_1' yield spread oscillates from 0.27 bps to 0.31 bps in "Straight Bonds" and "Payment Blockage" scenarios in **Table 2**. Note that significant oscillating pricing results like Fig. 1 demonstrated by Wang et al. (2014) can severely interfere our analyses. Fortunately, they alleviate the oscillating problem by making some tree nodes coincide with default boundaries and show that their pricing results converge smoothly with the increment of the number of time steps of their tree method. Our framework adopts their tree adjustment technique and generates stable numerical results. The fast and smoothly converging property can be verified by observing that all pricing results change mildly (less than 0.4%) when the `Time Steps` (listed in the second row) increases from 32 to 512. This good property allows us to analyze many empirical phenomena, says the existence of the payment blockage covenants on reducing the yield spreads of B_3, B_4, and B_5.

Scenario	Straight Bonds			Payment Blockage			Multiple Callable Bonds		
`Time Steps`	**32**	**128**	**512**	**32**	**128**	**512**	**32**	**128**	**512**
B_1(5yr)	103.85	103.86	103.85	103.86	103.86	103.86	103.86	103.85	103.85
	(0.30)	(0.29)	(0.31)	(0.29)	(0.27)	(0.30)	(0.30)	(0.34)	(0.31)
B_2(10yr)	110.62	110.66	110.68	107.99	107.96	107.96	104.20	104.24	104.22
	(43.24)	(42.73)	(42.43)	(76.55)	(77.01)	(77.02)	(126.51)	(125.88)	(126.21)
B_3(16yr)	127.36	127.30	127.34	128.24	128.20	128.25	104.55	104.81	104.91
	(98.59)	(99.10)	(98.81)	(91.12)	(91.41)	(91.06)	(322.66)	(319.72)	(318.69)
B_4(20yr)	138.83	138.80	138.75	139.71	139.70	139.66	143.23	143.08	143.04
	(115.36)	(115.58)	(115.94)	(109.10)	(109.14)	(109.44)	(84.44)	(85.48)	(85.77)
B_5(30yr)	180.16	180.15	180.13	181.03	181.05	181.04	186.75	186.58	186.55
	(121.52)	(121.59)	(121.66)	(117.31)	(117.25)	(117.28)	(90.57)	(91.35)	(91.47)
E	339.17	339.24	339.25	339.17	339.24	339.25	357.41	357.43	357.43

Table 2: Robustness Checks. Bond prices, corresponding bond yields (listed in parentheses), and equity values E (in the last row) generated under our framework are examined under three different scenarios denoted by the first row. Five \$100 outstanding bonds, B_1, B_2, B_3, B_4, and B_5 are assumed to be issued by the same hypothetical issuer with coupon rates 7%, 8%, 10%, 11%, and 14%, and the remaining time to maturities 5, 10, 16, 20, and 30 years, respectively. The yield spreads for the junior bond in the "Payment Blockage" scenario and callable bonds in the "Multiple Callable Bonds" scenario are colored in red. The current issuer's asset value V_0 is 1000, its volatility σ is 20%, and the risk-free rate r is 6%. `Time Steps` in the second row denotes the number of the time steps used to partition the time span of one year in our framework.

Appendix C The Relationships among the Yield Spread, the Interest Rate Level, and the Issuer's Financial Status

Besides generating a feasible upward-sloping yield spread curves, our framework can quantitatively analyze the impacts of the interest rate level, the firm value level and its volatility on the yield spread, and these relations are investigated in past empirical researches, like Duffee (1998) and Avramov et al. (2007). The former work documents the relationship between the bond yield spreads and the interest rate levels and further identifies how the bond issuing firm's creditworthiness and the existence of call options in bonds affect this relationship. The main empirical results are listed as follows:

(i) Bond yield spreads decrease with the interest rate level.

(ii) The relation in **(i)** is stronger for defaultable callable bonds.

(iii) For defaultable straight bonds, the relation in **(i)** is stronger for bonds with lower credit ratings.

(iv) For defaultable callable bonds, the relation in **(i)** is stronger for bonds when the interest rate level is low.

The latter work documents the relationship between the bond yield spreads and the firm value volatility and also identifies how the bond issuing firm's creditworthiness influences this relationship. Their empirical results are listed as follows:

(v) Bond yield spreads increase with the volatility of the firm's asset value.

(vi) For defaultable straight bonds, the relation in **(v)** is stronger for bonds with lower credit ratings.

Several hypothetical scenarios are quantitatively analyzed through our valuation framework, and the numerical results are displayed in **Fig. 14** and **Fig. 15** for comparing with the above empirical observations. Some patterns can also be captured by an extant structural models, like the Merton (1974)'s model. For example, in his model, a defaultable straight bond can be decomposed into an otherwise identical default-free bond minus a put option on the issuing firm's asset value. Thus, pattern **(i)** can be captured since the yield spread is positively related to the value of the put option, which decreases with the interest rate level. So does our valuation framework, and the numerical results for pattern **(i)** are displayed as the dashed curves in **Fig. 14**. Note that the difference between the black and gray dashed curves reflects the impacts of the difference in the bond maturities. The yield spread of the 16-year bond is greater than that of the 10-year bond since more insolvency burden is borne by the holder of the long-term 16-year bond, which is also consistent with the observation that the yield spreads increase with bond maturities as illustrated in **Fig. 1**.

The yield spread of a callable bond reflects the combination of the bond issuer's insolvency risk and the bond holders' call risk due to the issuer's premature redemption. When the interest rate level decreases, the issuer tends to redeem its callable bond (and issues another bond with lower coupon rate). This explains why the option adjusted spreads (`OAS`), which can be interpreted as the difference between solid (denote the spread for the callable bonds) and dashed curves (the spreads for otherwise identical straight bonds) in **Fig. 14**, decreases with the interest rate level. The solid and dashed curves overlap when the interest rate level is high, since the embedded call options are less likely to be exercised and thus the `OAS` converges to zero. Both the pattern **(ii)** and **(iv)** can now be theoretically analyzed: the decrease in the yield spread of a callable bond can be regarded as the decrease in the yield spread of the otherwise identical straight bond plus the decrease in the `OAS`. This spread-rate relationship can be quantitatively captured by the models treating an issuer's capital structure as a combination of equity and one bond, like Acharya and Carpenter (2002). So does our valuation framework that avoids such simplification, and the numerical results are displayed in **Fig. 14**. Based on this empirical validity, our framework allows us to further investigate how the wealth transfer among the equity and the remaining bond holders due to bond redemptions causes call delay phenomena discussed in later sections.

Fig. 15 displays yield spreads evaluated by our framework for the five bonds in the "Straight Bonds" scenario in the **Appendix B** to examine the above empirically observed patterns for straight bonds. **Fig. 15 (a)** exhibits the yield spread curves under different interest rate levels and the issuer's creditworthiness (proxied by its current asset value). The pattern **(i)** can be verified by observing that the bond yield spreads decrease from the dashed curves to solid ones regardless of the remaining time to bond maturities when the interest rate level increases from 2% to 6%. The deterioration in the issuer's creditworthiness, proxied by the decrement of the issuer's asset value from 2000 (plotted in the gray color) to 1000 (in the black color), would make the pattern **(i)** more significant, which is consistent with the observation **(iii)**. **Fig. 15 (b)** exhibits the yield spread curves under different issuer's creditworthiness and its asset value volatility which reflects the risk level to run a firm and is usually applied to analyze the asset substitution problem, like Leland (1994). The pattern **(v)** can be verified by observing that the bond yield spreads increase from solid curves to dash ones when the volatility increases from 20% to 25%. The decrement of the issuer's asset value from 2000 to 1000 would make the pattern **(v)** more significant, which is consistent with the observation **(vi)**. Based on this empirical validity, our framework further allows us to study how increment in the issuing firm's leverage affects the yield spreads of its existing bonds and why the bonds in the firm with greater proportion of short-term bonds that are about to be rolled over have high yield spreads.

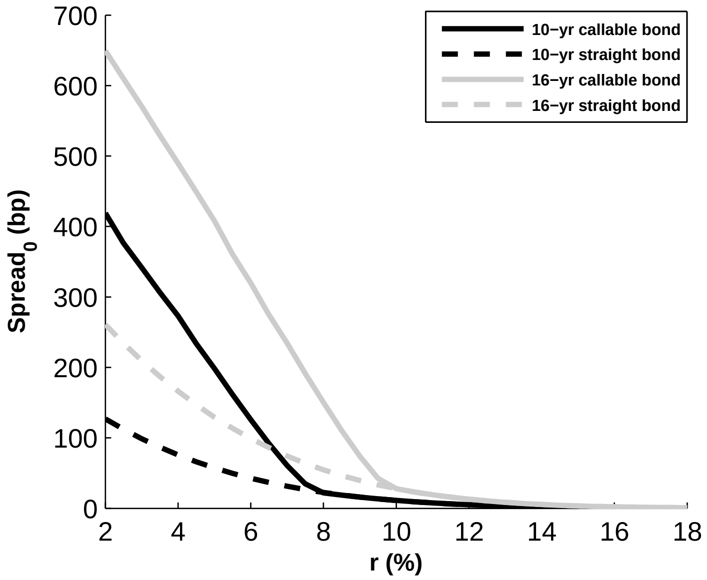


Figure 14: The relationship between the yield spread and the interest rate level. We here consider two otherwise identical scenarios same as those in Table 2 in Appendix B: the "Straight Bonds" and "Multiple Callable Bonds". The x- and the y- axes denote the interest rate level and the yield spread for the 10- and 16-year bonds, respectively. In the former scenario, the two bonds are straight bonds, and the yield spreads evaluated are plotted in dashed curves. In the latter scenario, the two bonds are callable bonds, and the yield spreads evaluated are plotted in solid curves. The tree and forest with 128 time steps per year are adopted. Except the interest rate level, all numerical settings are identical to those in Appendix B.

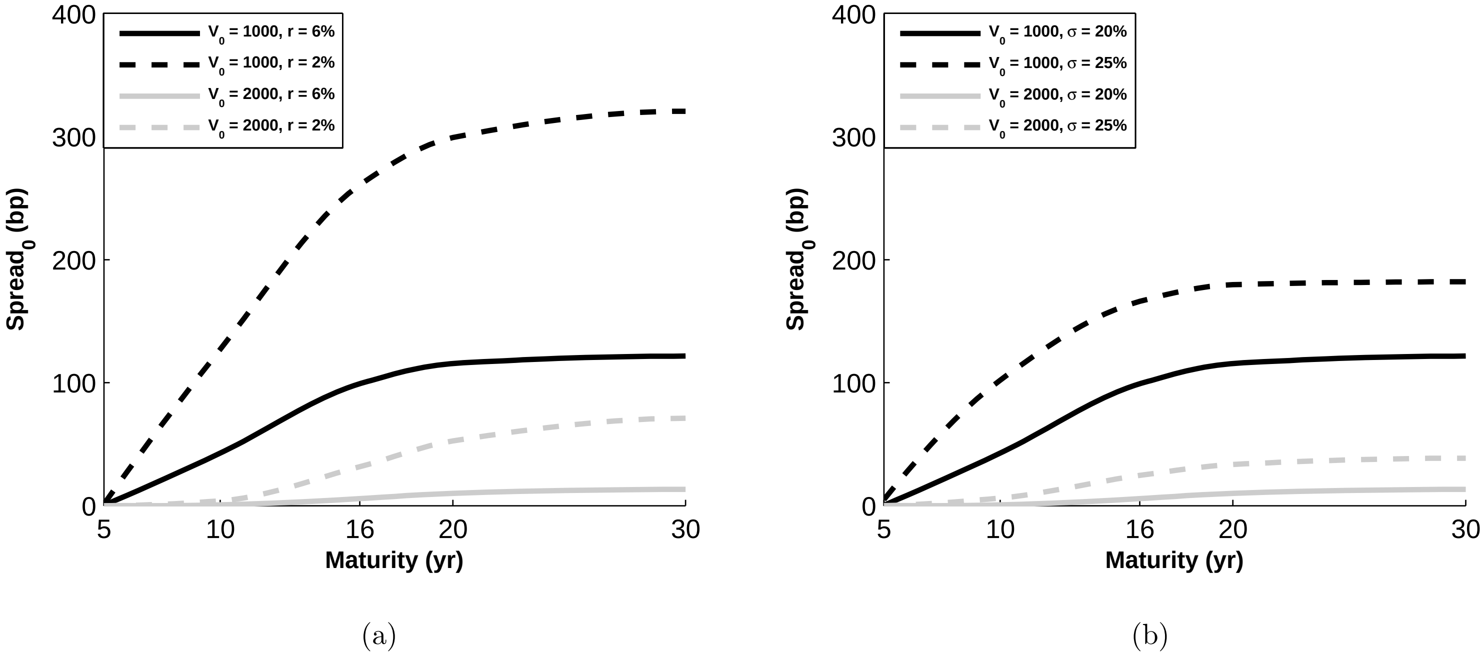


Figure 15: Yield spread curves under different scenarios. This figure displays the yield spread curves for the five bonds in "Straight Bonds" scenario in Table 2 in Appendix B under different interest rate levels, issuing firm's current asset values and the asset value volatility. The black and gray colors stand for the scenarios of the low asset value (1000) and the high one (2000), respectively. In panel (a), the solid and dash curves represent the scenarios of the high interest rate level (6%) and the low one (2%) when the asset value volatility is 20%, respectively. In panel (b), the dash and solid curves represent the scenarios of high asset value volatility (25%) and the low one (20%) when the interest rate level is 6%, respectively.